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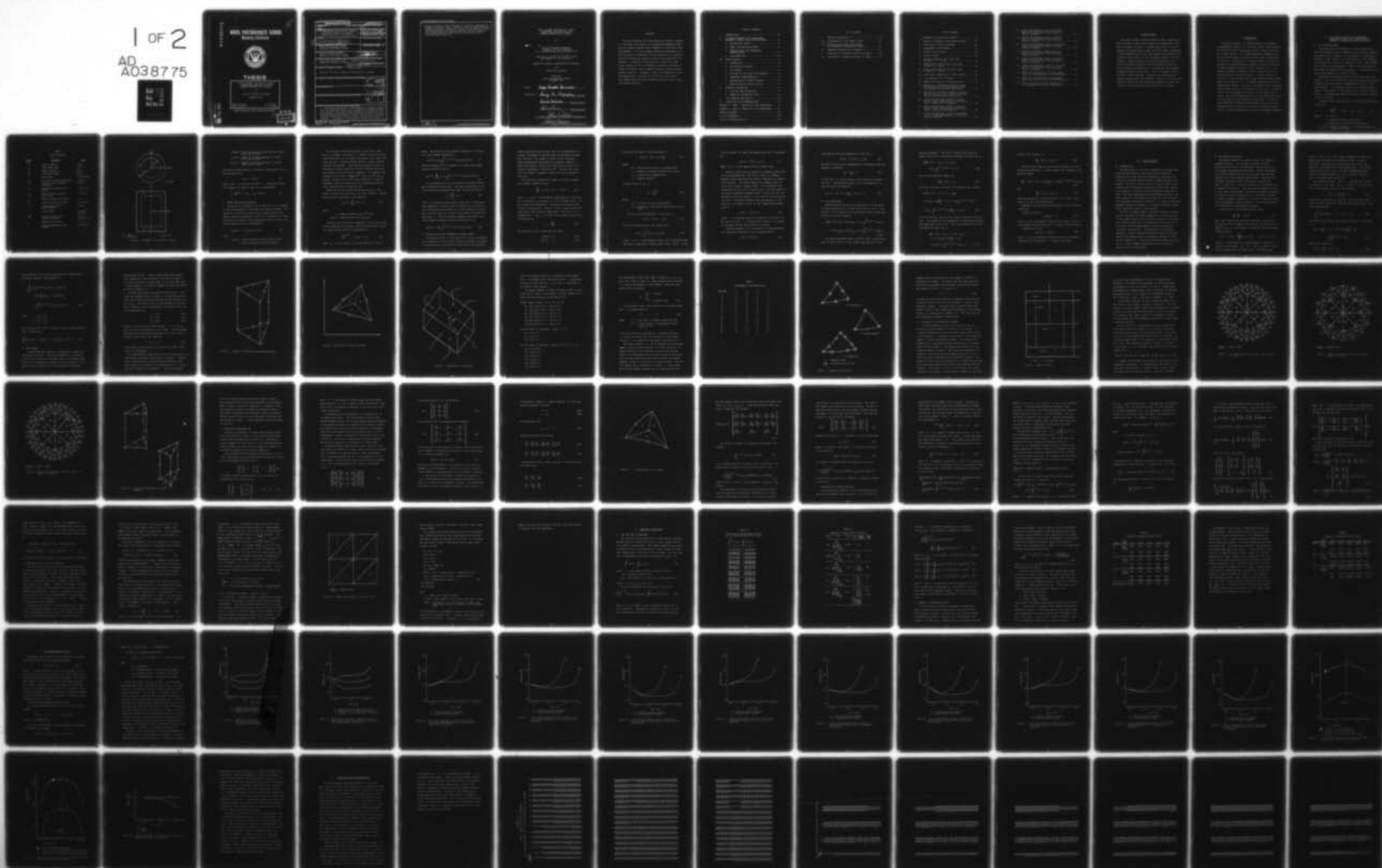

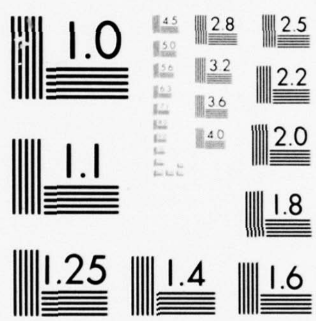
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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

FINITE ELEMENT SOLUTION OF A THREE-DIMENSIONAL NONLINEAR REACTOR DYNAMICS PROBLEM WITH FEEDBACK

by

Eulogio Conception Bermudes

December 1976

Thesis Advisor: D. Salinas
Thesis Advisor: D. H. Nguyen

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FINITE ELEMENT SOLUTION OF A THREE-
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DYNAMICS PROBLEM WITH FEEDBACK

by

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ABSTRACT

This work examines the three-dimensional dynamic response of a nonlinear fast reactor with temperature-dependent feedback and delayed neutrons when subjected to uniform and local disturbances. The finite element method was employed to reduce the partial differential reactor equation to a system of ordinary differential equations which can be numerically integrated. A program for the numerical solution of large sparse systems of stiff differential equations developed by Franke and based on Gear's method solved the reduced neutron dynamics equation. Although a study of convergence by refining element mesh sizes was not carried out, the crude finite element mesh utilized yielded the correct trend of neutron dynamic behavior.

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I. INTRODUCTION

The nuclear reactor with delayed neutrons and temperature-dependent feedback is a nonlinear system, whose response to uniform and local disturbances differs greatly from that of a linear reactor. The prompt temperature feedback model and one average group of delayed neutrons were incorporated in this work. Practically all neutron dynamics analysis today deals with two-dimensional geometry, implying a symmetry in neutron dynamics behavior during transient [1,2]. This symmetry assumption, however, could be unrealistic in the safety analysis of nuclear reactors. A unique feature of this work is the consideration of the three-dimensional dependence of the neutron flux. No symmetry assumptions were imposed on the problem. This work investigated neutron dynamics under uniform and local disturbances in the core. A uniform initial flux throughout the interior of the reactor was imposed. The finite element method (FEM) was employed to solve this nonlinear initial-boundary value problem. The FEM is quite effective in handling discontinuous forcing functions thereby making it particularly suited for examining the effects of localized perturbations and space-dependent feedbacks.

II. THE NUCLEAR REACTOR WITH TEMPERATURE DEPENDENT FEEDBACK AND DELAYED NEUTRONS

A. THE PHYSICAL SYSTEM

The system under consideration is a fast reactor of cylindrical geometry that is composed of two different regions. The reactor core or region I is cylinderical in shape and is fueled by U-235. Region II or the reflector region completely surrounds the core and is composed of U-238. Both regions were assumed to be homogeneous. Table I lists the physical properties and geometry for each region. A schematic of the fast reactor geomenry is shown in figure 1.

Temperature and delayed neutron effects were taken into account. Also, a one-velocity or one-group model was assumed, thereby making the velocity independent of spatial or temporal effects. The delayed neutrons were considered only in region I since region II was assumed to be a non-multiplying medium. The temperature effects were also assumed to be only in the core region.

In general form, the one-velocity neutron diffusion equation is [3]

$$\frac{\partial N(\bar{r},t)}{\partial t} = vD\nabla^2 N - \Sigma_a vN + S(\bar{r},t) \qquad (1)$$

where $\bar{r} = (x,y,z)$

D = neutron diffusion coefficient

Σ_a = macroscopic absorption cross-section

$N(\bar{r},t)dV$ = number of neutrons in a volume element dV at a point $\bar{r}$ at time t

TABLE I

Physical Constants

SYMBOL	DEFINITION	VALUE
R_I	radius of Region I	60 cm
R_{II}	total reactor radius	90 cm
H_I	height of Region I	160 cm
H_{II}	total reactor height	220 cm
v	neutron speed	4.8×10^7 cm/sec
D_I	core neutron diffusion coefficient	0.913 cm
D_{II}	reflector neutron diffusion coefficient	1.200 cm
Σ_{a_I}	core neutron absorption cross section	0.01401 cm^{-1}
$\Sigma_{\alpha_{II}}$	reflector neutron absorption cross section	0.0040 cm^{-1}
ν	number of neutrons per fission	2.54
Σ_f^*	critical fission cross section	0.005736 cm^{-1}
β	delayed neutron fraction; dollar reactivity	0.00642
ε	fission energy	7.652×10^{-12} cal/fission
$\bar{h}(\frac{A}{V})$	modified convection heat transfer coefficient	0.0632 cal/(cm^2sec $^\circ$C)
α	temperature coefficient	$-0.004/^\circ$C
$\bar{\lambda}$	abundance-weighted mean decay constant	0.4350 sec^{-1}

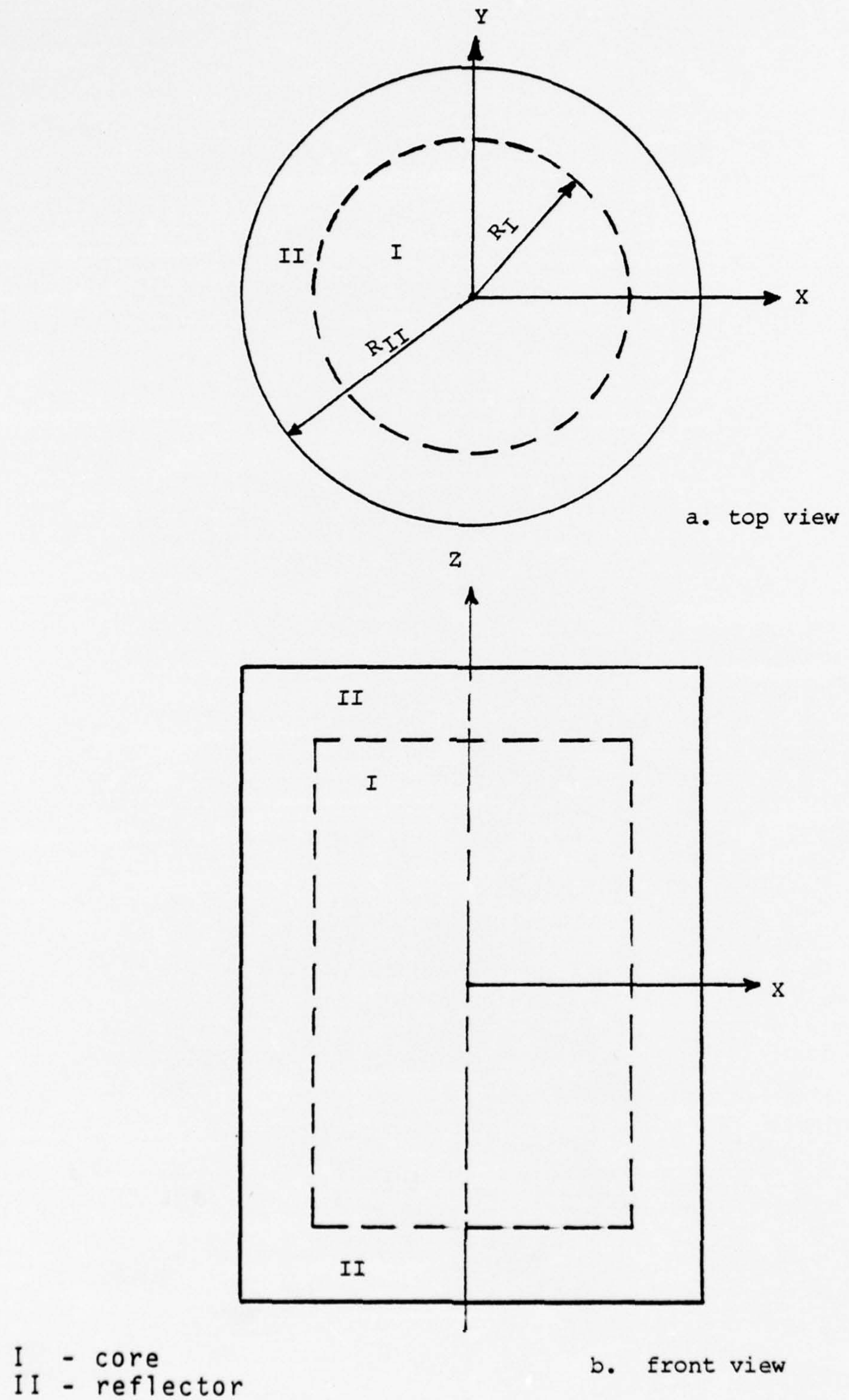


Figure 1. Schematic of cylindrical reactor.

$v D \nabla^2 N dV$ = number of neutrons diffusing into dV per unit time at time t

$\Sigma_a v N\, dV$ = number of neutrons absorbed in dV per unit time at time t

$S(\bar{r},t) dV$ = number of neutrons produced in dV per unit time at time t

The neutron number density is related to the neutron flux by the expression [4]

$$\phi(\bar{r},t) = vN(\bar{r},t) \tag{2}$$

where $\phi(\bar{r},t)$ is the flux at time t. The neutron diffusion equation in terms of the flux is depicted by

$$\frac{1}{v}\frac{\partial\phi(\bar{r},t)}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + S(\bar{r},t) \tag{3}$$

B. PROMPT AND DELAYED NEUTRONS

The source or production term in equation (3) is composed of the contributions of the prompt and delayed neutrons. The majority of the fission neutrons are prompt neutrons that appear almost instantaneously (within 10^{-7} second) on fission. Assuming a fast neutron non-leakage probability of unity, the prompt neutron source is described by

$$S_p(\bar{r},t) = (1-\beta)K_\infty(\bar{r},t)\ \phi(\bar{r},t) \tag{4}$$

where

$K_\infty(\bar{r},t)$ = infinite medium multiplication factor

β = total fraction of delayed neutrons

The fraction of delayed neutrons is very small (note that $\beta = 0.00642$ from Table I). However, they have a very significant effect on the reactivity because their mean lifetimes are long. Without delayed neutrons, reactor control would not be possible. These delayed neutrons are born in the decay by neutron emission of nuclei produced following the β-decay of certain fission fragments. For example, the β-decay of the fission fragment Br^{87} leads to Kr^{86} plus a neutron. Nuclei such as Br^{87} whose production in fission eventually leads to the emission of a delayed neutron are called delayed neutron precursors [4].

There are six main groups of delayed neutrons. Each group is classified according to its decay constant. The delayed neutron source term is portrayed by

$$S_d(\bar{r},t) = \sum_{i=1}^{6} C_i(\bar{r},t)\, \lambda_i \qquad (5)$$

where

$$\lambda_i = \text{decay constant of the } i^{th} \text{ group}$$
$$C_i(\bar{r},t) = \text{density of the } i^{th} \text{ precursor}$$

Assuming that the fission fragments do not migrate appreciable distances and assuming a non-circulating fuel reactor [3], the precursor density is delineated by

$$\frac{\partial C_i(\bar{r},t)}{\partial t} = \beta_i K_\infty \Sigma_a \phi - \lambda_i C_i \qquad (6)$$

where β_i is the fraction of delayed neutrons of the i^{th}

group. The solution to the precursor equation is in terms of a time integral expressed by

$$C_i(\bar{r},t) = \beta_i \Sigma_a \int_0^t e^{-\lambda_i(t-t')} K_\infty(\bar{r},t)\phi(\bar{r},t)dt' \qquad (7)$$

Inserting equation (7) in equation (5) yields the delayed neutron production term as

$$S_d(\bar{r},t) = \sum_{i=1}^{6} \beta_i \lambda_i \Sigma_a \int_0^t e^{-\lambda_i(t-t')} K_\infty(\bar{r},t)\phi(\bar{r},t)dt' \qquad (8)$$

For convenience the six main groups of delayed neutrons were considered as one group. This was accomplished by using the abundance-weighted mean decay constant defined by

$$\bar{\lambda} \approx \frac{1}{\beta} \sum_{i=1}^{6} \beta_i \lambda_i \qquad (9)$$

This is a reasonable approximation since many of the important phenomena of nuclear reactor dynamics can be characterized satisfactorily by combining all the emitters or precursors into one, two, or, at most, three effective groups [3]. Replacing λ_i with the abundance-weighted mean decay constant showed the delayed neutron source term to be

$$S_d(\bar{r},t) = \beta\bar{\lambda}\Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')} K_\infty(\bar{r},t)\phi(\bar{r},t)dt' \qquad (10)$$

C. DOPPLER EFFECT AND TEMPERATURE FEEDBACK MODEL

The Doppler effect in fast reactors is due to the temperature broadening of many closely spaced high-energy resonances in both the fission and parasitic-absorption cross-sections.

These resonances basically mean that as the temperature increases, the number of neutrons that are absorbed increases and, similarly, the number of fission events increases. These nonproductive and productive processes compete in a complicated manner, and the net effect may be either an increase or decrease in reactivity [3]. For most fast reactors, the effect is negative, and it will be so assumed in this work.

The reactivity change with respect to the fuel temperature change is modeled by [2]

$$\frac{dK_\infty}{dT} = aT^{-3/2} + bT^{-1} + cT^{(m-1)} \tag{11}$$

where a, b, and c are parameters determined from experimental or neutronic calculations, m is an integer, and T is the current fuel temperature. For most fast reactor cores (ceramic-fueled cores), TdK_∞/dT is very close to being constant over a wide range of temperatures. Therefore, it is assumed that a and c are zero and the Doppler coefficient is expressed by

$$b = T\frac{dK_\infty}{dT} \tag{12}$$

The following initial conditions were used:

$$K_\infty(\bar{r},0^+) = K_\infty^\circ \tag{12a}$$

$$T(\bar{r},0^+) = T_0 \tag{12b}$$

The reactivity model is then expressed by

$$K_\infty(\bar{r},t) = K_\infty^\circ + b \; \ell n \left(\frac{T}{T_o}\right) \tag{13}$$

where

K_∞° = multiplication factor at steady-state

ν = number of neutrons produced per fission

T_o = original fuel temperatures

b = Doppler constant

The definition of K_∞° is

$$K_\infty^\circ = \frac{\nu \Sigma_f^{\;*}}{\Sigma_a^{\;c}} \tag{14}$$

where

$\Sigma_f^{\;*}$ = critical fission cross-section

$\Sigma_a^{\;c}$ = macroscopic absorption cross-section of the core

The rise in fuel temperature is depicted by

$$\theta(\bar{r},t) = T(\bar{r},t) - T_o(\bar{r}) \tag{15}$$

This is also described by the integral [5]

$$\theta(\bar{r},t) = \int_0^t f(t-t') \; \psi(\bar{r},t)dt' \tag{16}$$

where $f(t-t')$ is the feedback kernel and is dependent upon the type of temperature feedback model used. $\psi(\bar{r},t)$ is the

rise in neutron flux above the steady state and is expressed by

$$\psi(\bar{r},t) = \phi(\bar{r},t) - \phi_0(r) \qquad (17)$$

where $\phi_0(r)$ is the neutron flux at steady state.

There are three types of temperature feedback models that can be considered here. The first is known as "Newton's feedback" which determines the reactor temperature by Newton's law of cooling. The second temperature feedback model is called the adiabatic feedback model. This represents the temperature for the loss of coolant case. The third model is the prompt temperature feedback model in which the fuel temperature follows the behavior of the neutron flux without delay [5,6]. The prompt feedback model was employed in this analysis. The feedback kernel for this temperature feedback model is

$$f(t-t') = \frac{K}{\gamma}\,\delta(t-t') \qquad (18)$$

where K is an energy production operator with units of °C per unit flux and γ, a dimensionless quantity, is related to the mean time for heat transfer to coolant.

Inserting equation (18) in equation (16) and performing the integration produced a rise in temperature of

$$\theta(\bar{r},t) = \frac{K}{\gamma}\,\psi(\bar{r},t) \qquad (19)$$

From equation (19) the temperature of the fuel is

$$T(\bar{r},t) = \frac{K}{\gamma}\,\psi(\bar{r},t) + T_o(\bar{r}) \tag{20}$$

The ratio of the current temperature to the steady-state temperature is therefore

$$\frac{T}{T_o} = \frac{K}{\gamma T_o}\,\psi + 1 \tag{21}$$

In this work T_o was considered to be constant throughout the core. Incorporating equation (21) into equation (13) gave the reactivity model of

$$K_\infty(\bar{r},t) = K_\infty^\circ + b\,\ell n\left[\frac{K}{\gamma T_o}\,\psi(\bar{r},t) + 1\right] \tag{22}$$

D. FIELD EQUATIONS

Before establishing the field equations it is desirable to express equation (3) in terms of the rise in flux. Using equation (17) in equation (3) and grouping terms yielded the following diffusion equation:

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = \left[D\nabla^2\psi - \Sigma_a\psi + (1-\beta)K_\infty\Sigma_a\psi + \beta\bar{\lambda}\Sigma_a\int_0^t e^{-\bar{\lambda}(t-t')}K_\infty\psi dt'\right]$$

$$+ \left[D\nabla^2\phi_o - \Sigma_a\phi_o + (1-\beta)K_\infty\Sigma_a\phi_o + \beta\bar{\lambda}\Sigma_a\int_0^t e^{-\bar{\lambda}(t-t')}K_\infty\phi_o dt'\right] \tag{23}$$

The second bracketed term in equation (23) is identically equal to zero since it is the steady state portion of the

diffusion equation. The rise in neutron flux above its steady state value is therefore expressed, for the core, by

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = D\nabla^2\psi - \Sigma_a\psi + (1-\beta)\ K_\infty\Sigma_a\psi$$

$$+\ \beta\bar{\lambda}\Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')}\ K_\infty\psi\ dt' \qquad (24)$$

and, for the reflector region, by

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = D\nabla^2\psi - \Sigma_a\psi \qquad (25)$$

Inserting the reactivity model into equation (24) yielded

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = D\nabla^2\psi - \Sigma_a\psi + (1-\beta)\ \Sigma_a K_\infty^\circ\psi$$

$$+(1-\beta)b\Sigma_a\ \{\ell n[\frac{K\psi}{\gamma T_o} + 1]\}\ \psi + \beta\bar{\lambda}K_\infty^\circ \int_0^t e^{-\bar{\lambda}(t-t')}\psi dt'$$

$$+\ \beta\bar{\lambda}\Sigma_a b\ \{\int_0^t e^{-\bar{\lambda}(t-t')}[\ell n(\frac{K\psi}{\gamma T_o} + 1)]\ \psi dt'\} \qquad (26)$$

For the reflector, the last four terms of equation (26) are zero. The effects of the temperature on the delayed neutrons were neglected in this work. The field equations can now be expressed, for the core, as

$$\frac{\partial\psi}{\partial t} - vD\nabla^2\psi + [v\Sigma_a - v(1-\beta)\ \nu\Sigma_f]\psi$$

$$+\ [-(1-\beta)v\Sigma_a b]\ [\ell n(\frac{K\psi}{\gamma T_o} + 1)]\psi$$

$$+\ [-\beta\bar{\lambda}v\Sigma_f\nu]\ [\int_0^t e^{-\bar{\lambda}(t-t')}\psi dt'] = 0 \qquad (27)$$

and, for the reflector, as

$$\frac{\partial\psi}{\partial t} - vD\nabla^2\psi + v\Sigma_a\psi = 0 \tag{28}$$

The non-linear terms of the core field equation will be linearized accordingly. In more compact form, equations (27) and (28) became

$$\frac{\partial\psi}{\partial t} - c1\nabla^2\psi + c2\psi + c4[\ell n(\frac{K\psi}{\gamma T_o} +1)]\psi + c5[\int_0^t e^{-\bar{\lambda}(t-t')}\psi dt'] = 0 \tag{29}$$

and

$$\frac{\partial\psi}{\partial t} - c1\nabla^2\psi + c2\psi = 0 \tag{30}$$

where the meanings of the coefficients c1, c2, etc., are obvious for the core and reflector.

Equations (29) and (30) were subjected to the following conditions:

boundary condition:

$$\psi_R(\bar{r}_B,t) = 0 \tag{31}$$

where $\bar{r}_B$ are coordinates of points on the outer surface of the reflector and the subscript R refers to the reflector

continuity of flux:

$$\psi_R(\bar{r},t) = \psi_c(\bar{r}_I,t) \tag{32}$$

where $\bar{r}_I$ are coordinates of points on the core-reflector interface and the subscript c refers to the core.

III. FINITE ELEMENT

A. INTRODUCTION

The application of the finite elements of nonlinear continua has been mostly in the field of solid mechanics. Prior work [1] using the finite elements of nonlinear continua on a nonlinear reactor dynamics problem has been successful. The FEM in this work was utilized to reduce a nonlinear partial differential equation of the nuclear reactor to a system of nonlinear ordinary differential equations in time. The time integration was accomplished by using a computer program for the numerical solution of stiff differential equations developed by Franke [7]. In order to minimize computer storage requirements, an optimum compacting scheme (OCS), described by Ref. 8, was adopted.

The finite element models of operator equations are generally classified into three categories: a) the variational finite element models such as the Ritz method, b) the weighted residuals method such as the method of Galerkin, and c) the direct finite element models which are not based on functional minimization. From experience in structural mechanics, the most effective method for generating acceptable finite element models of nonlinear equations is the Galerkin method [1]. This work adopted the method of Galerkin in a finite element approximation over the spatial domain of the field equations.

B. THE METHOD OF GALERKIN

The Galerkin method is a special case of the method of weighted residuals. It involves a rational choice of weighting function that is consistent with the type of finite element approximation considered. Indeed, the weighting functions chosen are the basis or shape functions employed in the finite element approximation. There are two favorable characteristics of the Galerkin method which makes it attractive. The first attribute is its amenability to integration by parts. This supplied the freedom of using a lower order finite element than might be otherwise possible. The second favorable characteristic of the Galerkin method is that the symmetric operators in the field equations transform into symmetric matrix operators. Both these attributes are attractive for computational purposes.

Consider the initial-boundary-value problem

$$\frac{\partial \psi}{\partial t} = \mathcal{L}\psi - f(\bar{r},t) \tag{33}$$

where $\mathcal{L}$ contains the nonlinear operators. According to the spatial finite element discretization, the solution of equation (33) is in the form of an union of an $\bar{N}$-term approximation given by

$$\psi(\bar{r},t) \simeq \tilde{\psi}(\bar{r},t) = \bigcup_{J=1}^{\bar{N}} \psi_J(t)\ G_J(\bar{r}) \ , \quad J=1,2,\ldots,\bar{N} \tag{34}$$

where $\bar{N}$ is the number of system degrees of freedom (i.e., number of coordinates), and $G_J(\bar{r})$ are the system or global basis functions which span the space of the approximate

solution $\tilde{\psi}(\bar{r},t)$ [1]. The Einstein summation is used. The global basic functions are "pyramid functions", each of which has a prescribed functional description over a subdomain of the system and is zero elsewhere [9]. The unknown coordinate functions $\psi_J(t)$ are the time-dependent magnitudes of the approximated flux $\tilde{\psi}(\bar{r},t)$ and/or its derivatives at discrete nodal points [10].

The residual function, $R(\bar{r},t)$, is defined such that it is identical to zero when $\tilde{\psi}(\bar{r},t)$ is equal to the exact solution. The residual function is expressed by

$$R(\bar{r},t) = \frac{\partial\tilde{\psi}}{\partial t} - \mathcal{L}\tilde{\psi} - f \qquad (35)$$

The Galerkin orthogonality condition (using the basis functions as weight functions), when applied to the residual function, requires that

$$\int_{Vol} G_I R(\bar{r},t)dVol = 0 \ , \quad I=1,2,\ldots,\bar{N} \qquad (36)$$

From the field equations, the residual function for the core is

$$R(\bar{r},t) = \frac{\partial\tilde{\psi}}{\partial t} - c1\nabla^2\tilde{\psi} + c2\tilde{\psi} + c4[\ln(\frac{K\tilde{\psi}}{\gamma T_o} + 1)]\tilde{\psi}$$

$$+ c5[\int_0^t e^{-\bar{\lambda}(t-t')}\tilde{\psi}dt'] \qquad (37)$$

and for the reflector

$$R(\bar{r},t) = \frac{\partial\tilde{\psi}}{\partial t} - c1\nabla^2\tilde{\psi} + c2\tilde{\psi} \qquad (38)$$

Using equation (34) and applying Galerkin's orthogonality condition produced a core equation of

$$\int_{Vol} G_I\{G_J\dot{\psi}_J(t)-\nabla^2 G_J(c1\cdot\psi)_J + G_J(c2\cdot\psi)_J$$

$$+[\ell n(\frac{K}{\gamma T_o} G_K\psi_K + 1)]\, G_J(c4\cdot\psi)_J$$

$$+[\int_0^t e^{-\bar{\lambda}(t-t')}(c5\cdot\psi)_J\, dt']G_J\} = 0 \qquad (39)$$

where $I = 1,2,\ldots,\bar{N}$

$J = 1,2,\ldots,\bar{N}$

$K = 1,2,\ldots,\bar{N}$

The reflector has a similar equation without the nonlinearity and is expressed by

$$\int_{Vol} G_I\{G_J\dot{\psi}_J(t) - \nabla^2 G_J(c1\cdot\psi)_J + G_J(c2\cdot\psi)_J\}\, dVol = 0 \qquad (40)$$

C. THE ELEMENT

A three-dimensional quadratic isoparametric element was employed in this work. The parent element is a triangular prism or solid wedge with straight sides. The element shape functions are expressed in terms of area coordinates in the plane of the triangle and by an isoparametric coordinate

along the prism axis. Figure 2 shows the parent element. This element was chosen because of the ease with which it fits the cylindrical structure when it is transformed into a curved element. This type of element has been used before as filler elements [11].

The area coordinates are defined by area ratios. Consider the triangle shown in figure 3. An arbitrary point P within the triangle defines three subareas designated by A_1, A_2, and A_3. The ratio of each of the subareas to the total area is known as an area coordinate. In equation form, the area coordinates are

$$L_1 = A_1/A \tag{41a}$$

$$L_2 = A_2/A \tag{41b}$$

$$L_3 = A_3/A \tag{41c}$$

where A is total area of the triangle. L_1, L_2, and L_3 are the natural coordinates for a triangle. The requirement that the sum of the subareas be equal to the total area is obviously satisfied by the identity

$$L_1 + L_2 + L_3 = 1 \tag{42}$$

In the plane of the triangle, only two of the area coordinates are independent.

The isoparametric coordinates are best visualized by considering the rectangular prism shown in figure 4. Isoparametric coordinates are normalized coordinates such that their values on the faces of the rectangle are $\pm$ 1. The $\xi\eta\zeta$ axes are in general not orthogonal. They are orthogonal

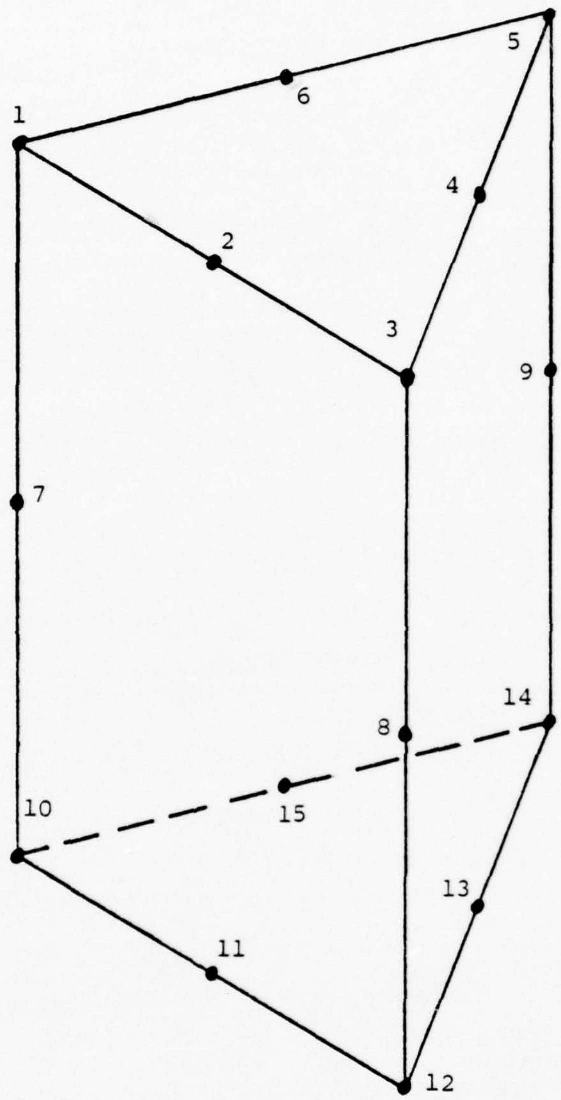


Figure 2. Quadratic triangular prism parent element

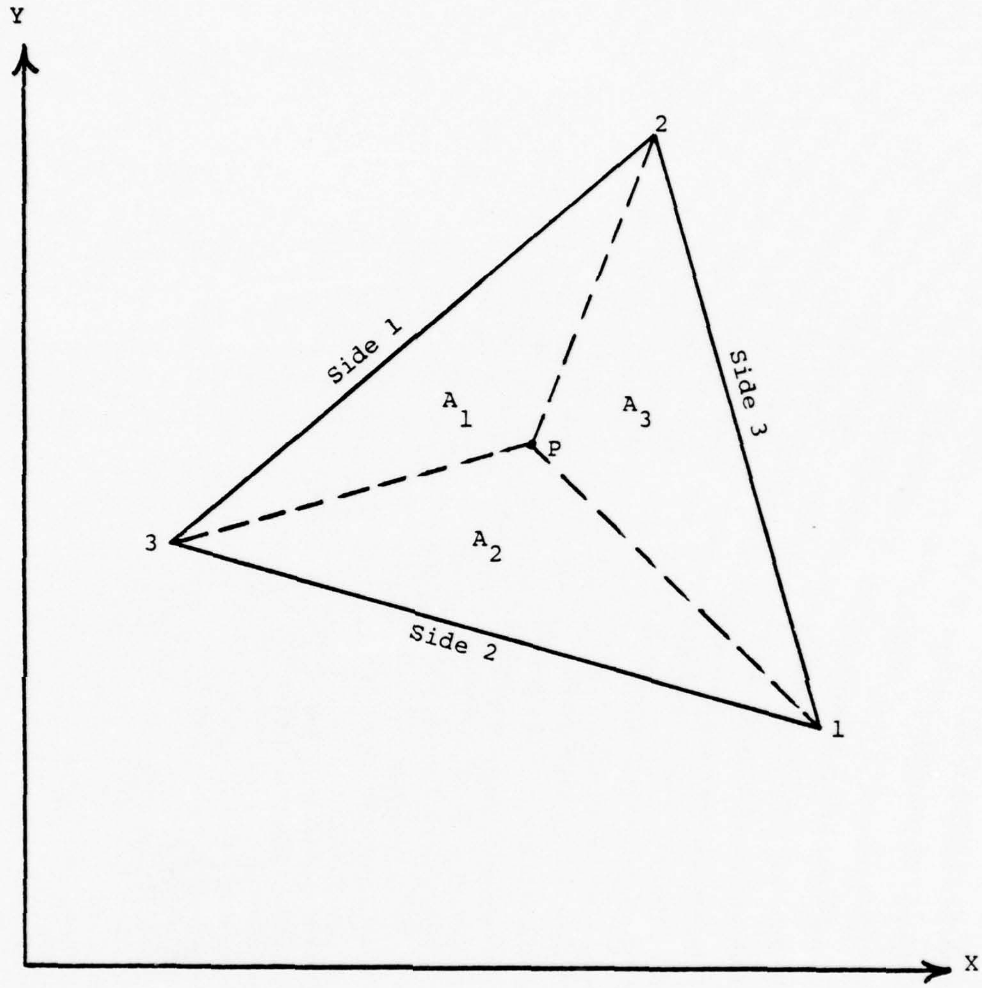


Figure 3. Definition of area coordinates

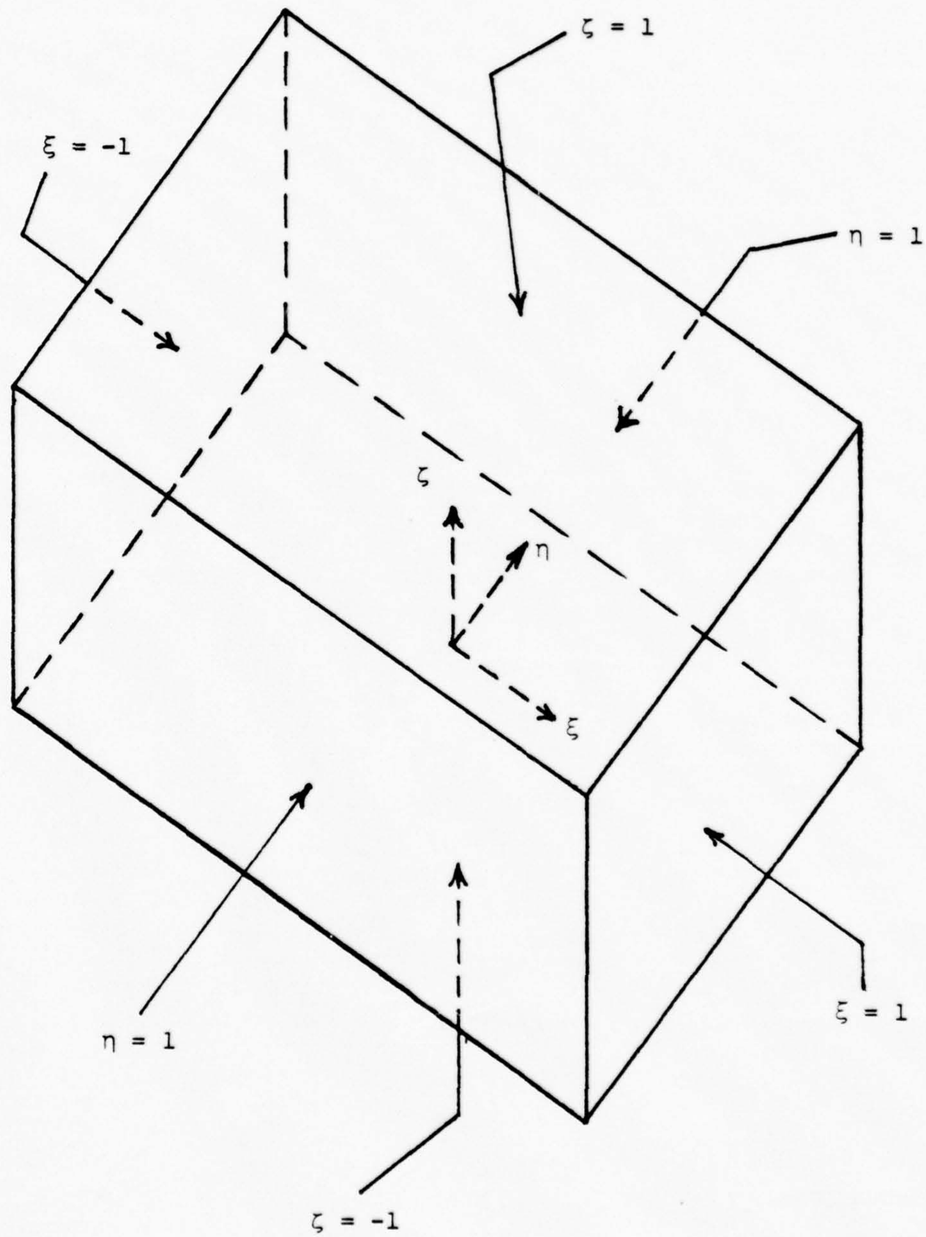


Figure 4. Isoparametric coordinates

only in the special case of a rectangular prism element [12]. The element basis functions use the ζ coordinate in the prism axis and the L_1, L_2, and L_3 coordinates in the plane of the triangle.

The parent element, as shown in figure 2, has 15 local nodal points around its periphery. As such, there are 15 basis functions which are given below [11]:

Corner nodes: (nodes 1, 3, 5, 10, 12, 14)

$$N_1 = \frac{1}{2} L_1(2L_1-1)(1+\zeta) - \frac{1}{2} L_1(1-\zeta^2)$$
$$N_3 = \frac{1}{2} L_2(2L_2-1)(1+\zeta) - \frac{1}{2} L_2(1-\zeta^2)$$
$$N_5 = \frac{1}{2} L_3(2L_3-1)(1+\zeta) - \frac{1}{2} L_3(1-\zeta^2)$$
$$N_{10} = \frac{1}{2} L_1(2L_1-1)(1-\zeta) - \frac{1}{2} L_1(1-\zeta^2)$$
$$N_{12} = \frac{1}{2} L_2(2L_2-1)(1-\zeta) - \frac{1}{2} L_2(1-\zeta^2)$$
$$N_{14} = \frac{1}{2} L_3(2L_3-1)(1-\zeta) - \frac{1}{2} L_3(1-\zeta^2)$$

Midside nodes of rectangles: (nodes 7, 8, 9)

$$N_7 = L_1(1-\zeta^2)$$
$$N_8 = L_2(1-\zeta^2)$$
$$N_9 = L_3(1-\zeta^2)$$

Midside nodes of triangles: (nodes 2, 4, 6, 11, 13, 15)

$$N_2 = 2L_1L_2(1+\zeta)$$
$$N_4 = 2L_2L_3(1+\zeta)$$
$$N_6 = 2L_3L_1(1+\zeta)$$
$$N_{11} = 2L_1L_2(1-\zeta)$$
$$N_{13} = 2L_2L_3(1-\zeta)$$
$$N_{15} = 2L_3L_1(1-\zeta)$$

The coordinates of each local node, in terms of L_1, L_2, L_3, and ζ are listed in table II. These element basis functions $< N >$ define the geometry of the element. Note that they satisfy the relationship

$$N_i = \begin{cases} 1 \text{ , at node i} \\ \\ 0 \text{ , at other nodes} \end{cases} \tag{43}$$

On the element level, the variation of the unknown function ψ is approximated by

$$\tilde{\psi}^e = < N' > \{\psi\}^e \tag{44}$$

where $< N'>$ = row vector of element shape functions

$\{\psi\}^e$ = column vector of time-dependent nodal values of $\tilde{\psi}^e$

To satisfy continuity requirements, the shape functions $< N'>$ have to be such that the continuity of the unknown function ψ is preserved in the parent coordinates [11].

The shape functions $< N >$ which characterize the element geometry and the shape functions $< N'>$ which describe the unknown function do not necessarily have to be the same. There is no requirement that the nodal values be associated with the same nodes which were used to define the element geometry, though in practice it is often the case. Consider for example, the illustrations in figure 5, If the nodes defining the element geometry and the nodes defining the

TABLE II

Coordinates of Local Nodal Points

Local Node	L_1	L_2	L_3	5
1	1	0	0	1
2	½	½	0	1
3	0	1	0	1
4	0	½	½	1
5	0	0	1	1
6	½	0	½	1
7	1	0	0	0
8	0	1	0	0
9	0	0	1	0
10	1	0	0	-1
11	½	½	0	-1
12	0	1	0	-1
13	0	½	½	-1
14	0	0	1	-1
15	½	0	½	-1

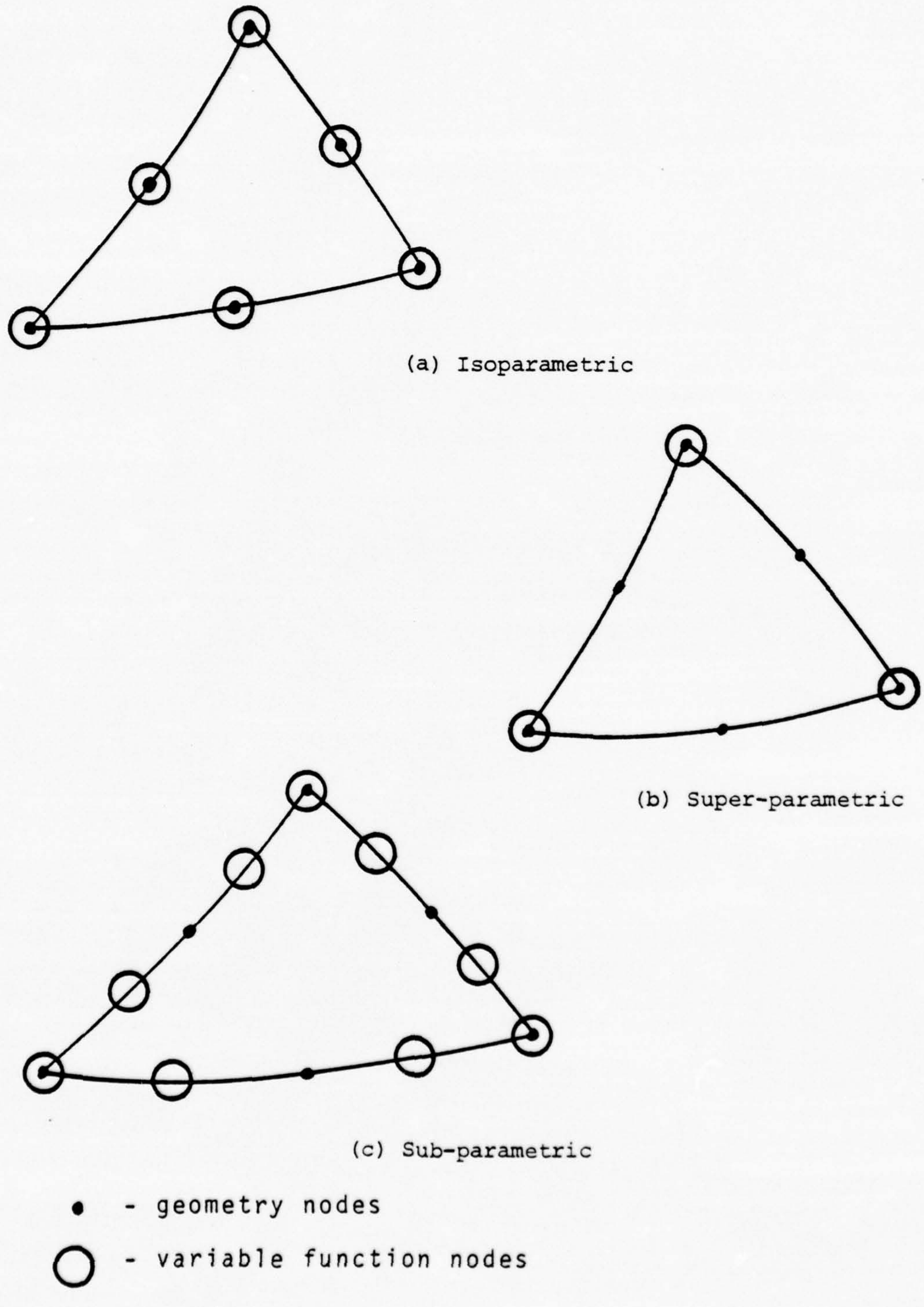


Figure 5. Element classification

unknown function are identical, the element is known as an isoparametric element. This means that the shape functions describing the geometry and the shape functions describing the unknown function ψ are equal, or

$$< N' > = < N > \tag{45}$$

If there are more nodes defining the geometry than nodes defining the variable function, the element is called a superparametric element. Using more nodal points to define the unknown function than to describe the geometry of the element results in a subparametric element [11]. This work utilized the isoparametric element classification.

D. DIVISION OF THE SYSTEM INTO ELEMENTS

In three-dimensional space, the division of the system into discrete elements is difficult to visualize. It is virtually impossible to show every nodal point of the system in one schematic. To present a clear view of the discretized domain, a "layer" approach was adopted. The first finite element grid or mesh employed here consisted of 128 elements. Under this grid (mesh I), the reactor was divided into four layers as shown in figure 6. Each layer was composed of 32 elements. The first and fourth layers were each 30 cm in height and each contained entirely reflector elements. The second and third layers were each 80 cm in height and together they encompassed the entire core plus the remaining reflector elements. Each layer, in turn, was partitioned into three horizontal (xy) planes. The top plane included all the global

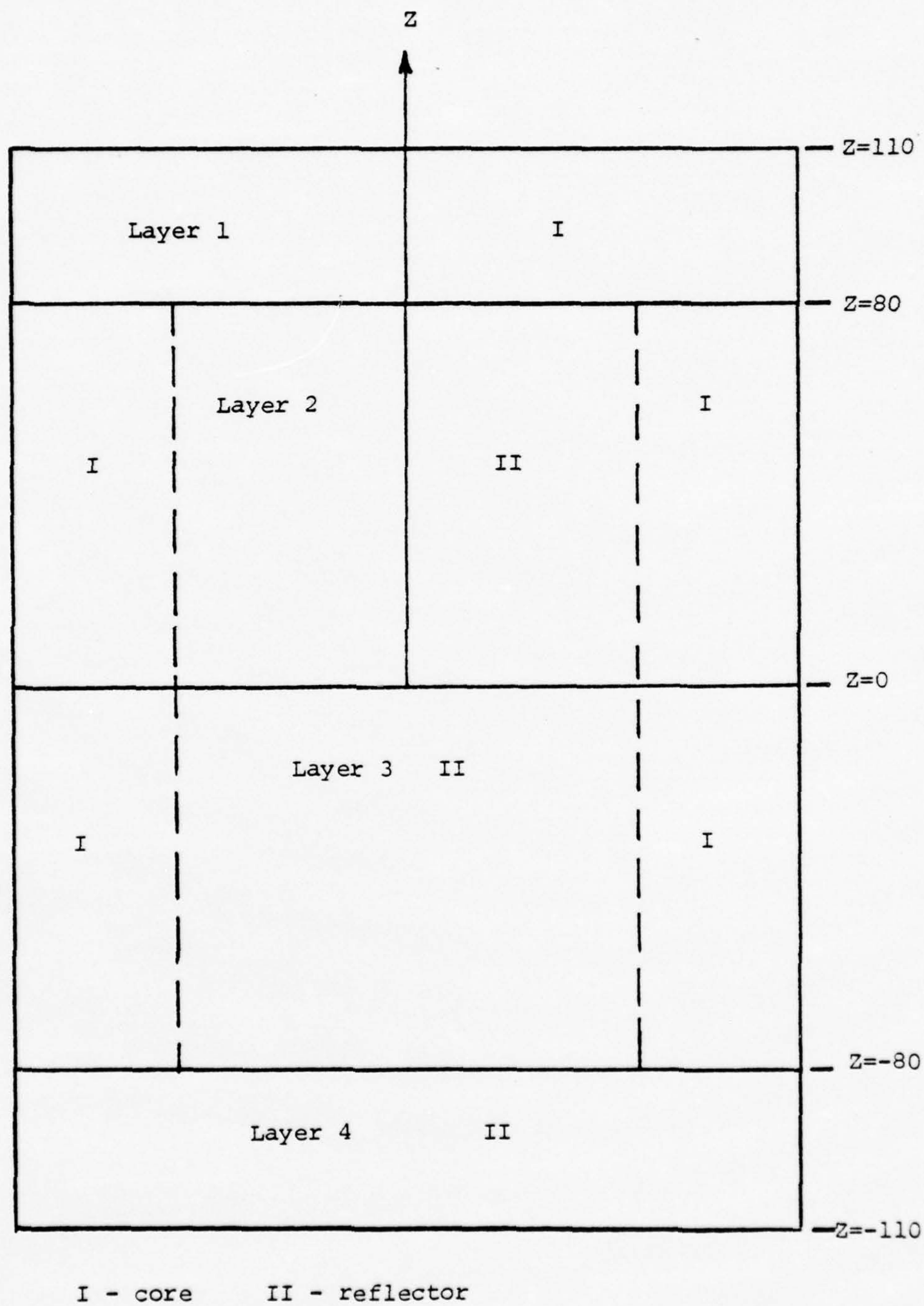


Figure 6. Layers of mesh I

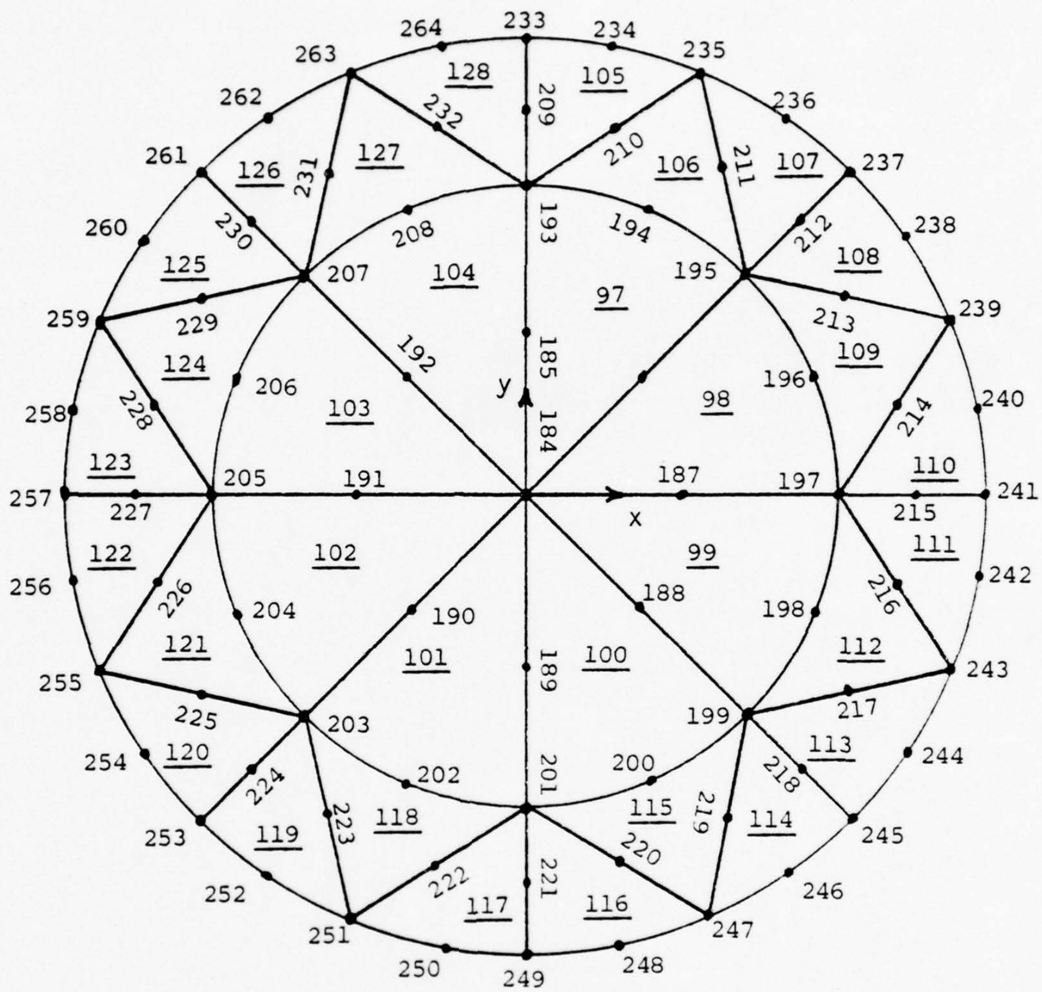
or system nodes corresponding to local or element nodes 1 through 6. The middle plane contained all the system nodes corresponding to the local nodes 7, 8, and 9. System nodes corresponding to element nodal points 10 through 15 comprised the bottom plane. To fix ideas, the three nodal planes of the first layer of mesh I are shown in figures 7, 8, and 9.

In this work there was only one curved side which was in the plane of the triangle, as shown in figure 10. The first, seventh, and tenth element nodes were each arbitrarily assigned to have as an opposite side the curved side of the triangle. The remaining local nodes in each of the respective planes of the element were then numbered consecutively in the counter-clockwise direction.

At this point it is appropriate to define connectivity. The connectivity of an element is a row vector array that relates the local nodes to system nodal points. The connectivity lists the system nodal points that are "connected" to form the element domain in the sequence of local nodal numbering. Thus, the connectivity matrix for mesh I is a matrix of size 128 x 15. To illustrate, the connectivity of element number 106 is

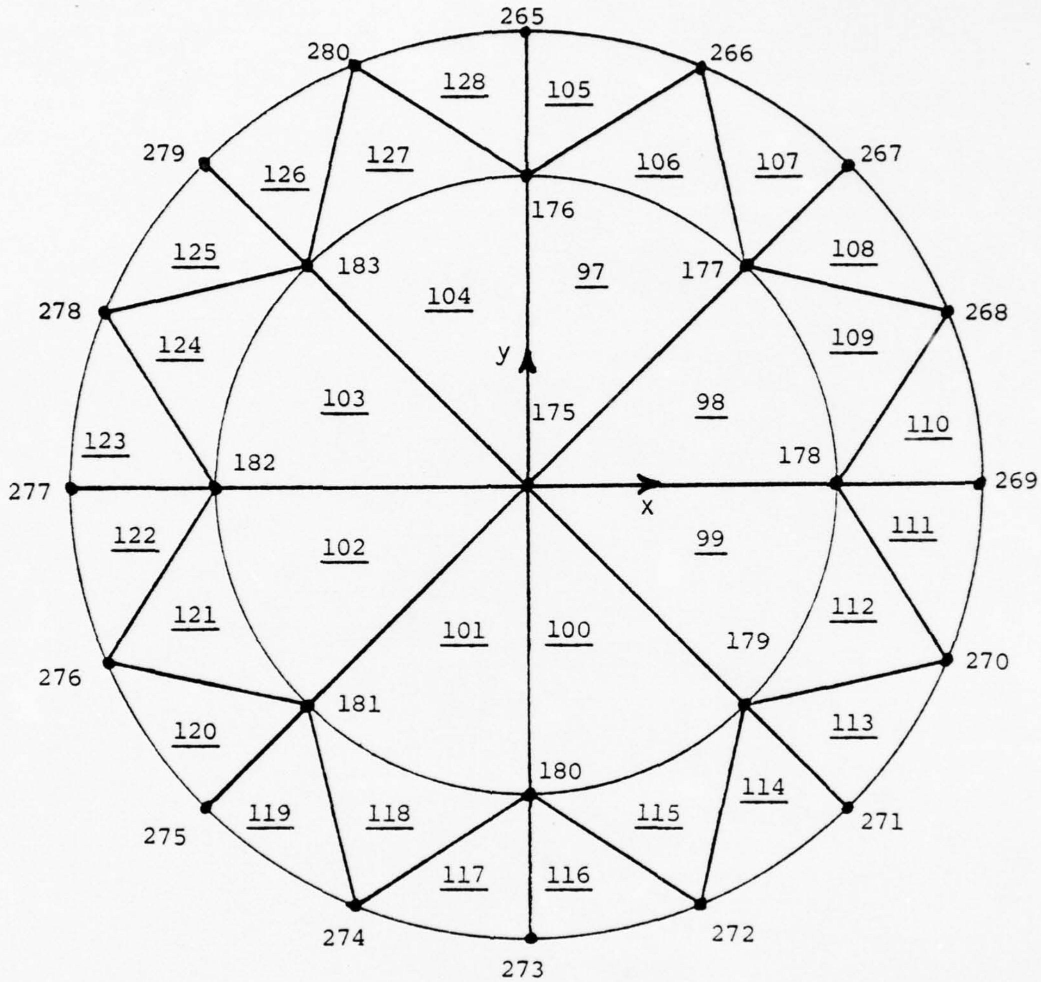
$$\langle 235, 210, 193, 194, 195, 211, 266, 176, 177, 283, 152, 10, 11, 12, 53\rangle$$

A second finite element mesh (mesh II) consisting of 192 elements was developed. It contained the same number of elements per layer as mesh I. However, mesh II has six layers. The second and third layers of mesh I were each divided in



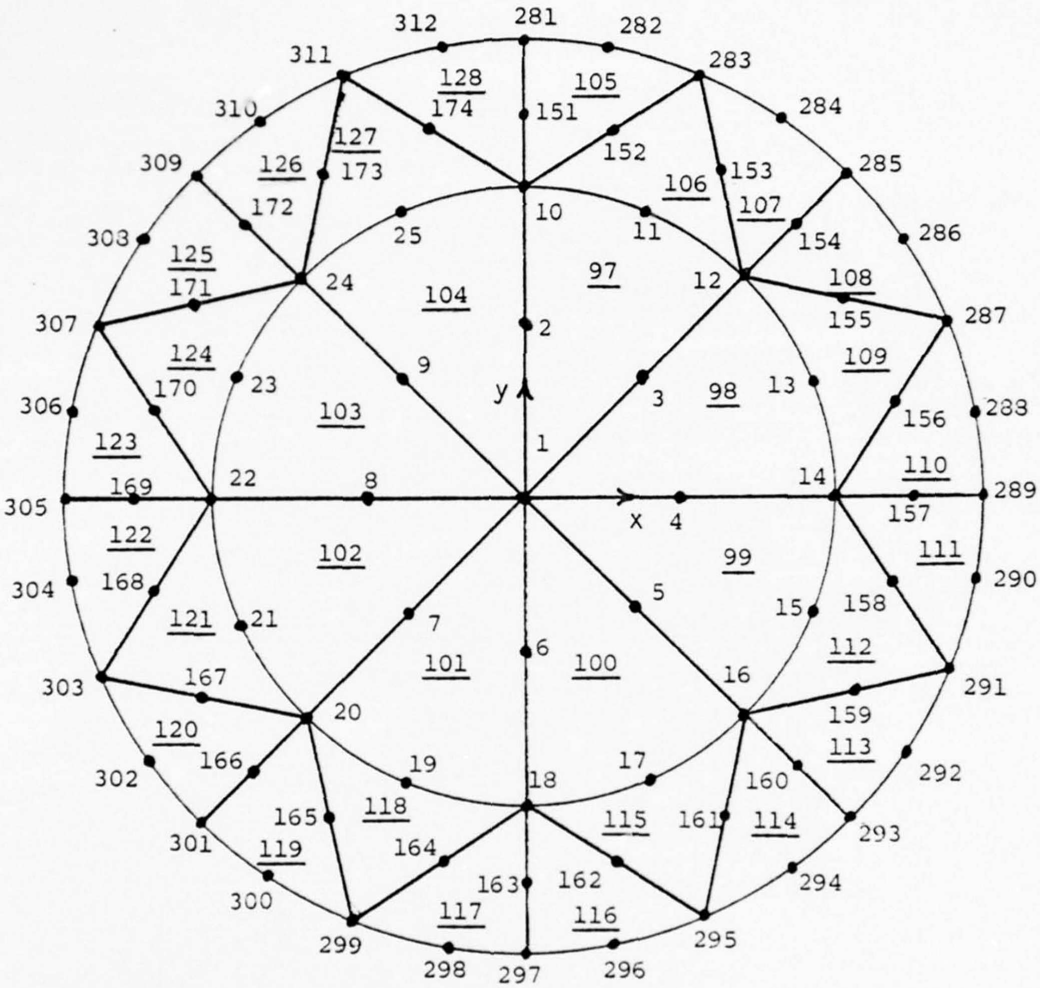
Number - element number

Figure 7. Top nodal plane of the first layer of mesh I, z = 110 cm



Number - element number

Figure 8. Middle nodal plane of the first layer of mesh I, z = 95 cm



Number - element number

Figure 9. Bottom nodal plane of the first layer of mesh I, z = 80 cm

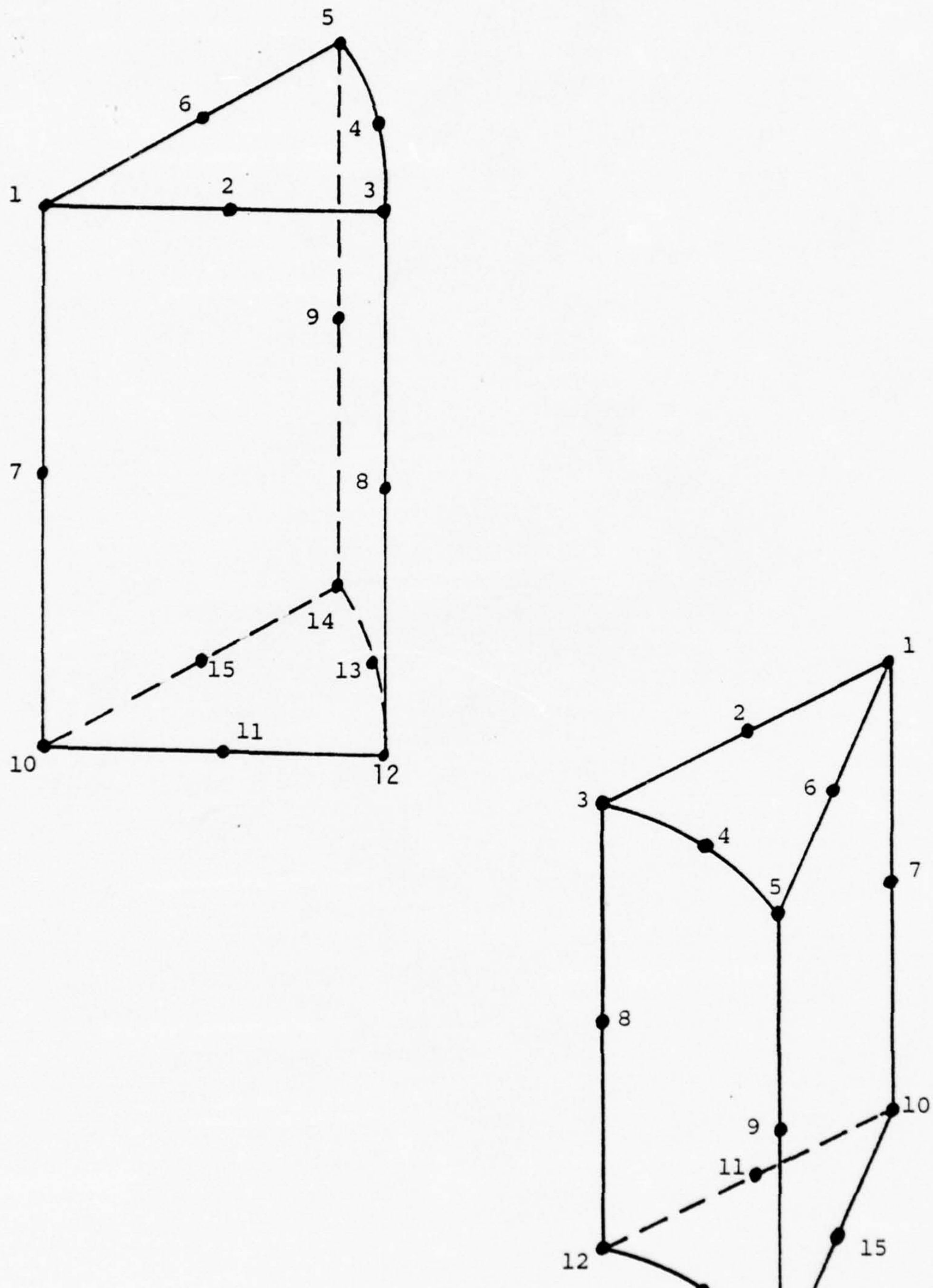


Figure 10. Local nodal numbering of curved elements

half, thus forming the two additional layers of mesh II. The connectivity matrix and the coordinates of each global node for mesh I are given in Appendix A. Appendix B lists the connectivity matrix and nodal coordinates of mesh II. The reader can construct the different layers and elements without much difficulty by using the nodal coordinates and the connectivity matrix. A mesh generator was not utilized in this work.

E. COORDINATE TRANSFORMATION

The use of a quadratic or higher order element permits the transformation or mapping of the straight-sided parent element into an element with curved sides. Distorted or curved elements provide a better fit to curved domains than linear elements, and thus a smaller number of elements is required to represent the structure adequately.

The transformation from cartesian coordinates to curvilinear coordinates can be accomplished by employing a one-to-one correspondence defined by [11]

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = f \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix} \quad \text{or } f \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \\ \zeta \end{Bmatrix} \qquad (46)$$

The element shape functions are utilized to achieve this transformation via the relation

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \sum_i N_i x_i \\ \sum_i N_i y_i \\ \sum_i N_i z_i \end{Bmatrix} \quad , \quad i=1,2,\ldots n^e \qquad (47)$$

where n^e is the number of element nodes, and the element shape functions N_i are in terms of local coordinates. Each set of local coordinates corresponds to only one set of cartesian coordinates.

In performing the transformation, the compatibility requirement must be met. The transformation into the new, curved elements should leave no gaps between adjacent elements. If two adjacent elements are generated from parents in which the element shape functions satisfy continuity requirements, then the curved elements will be contiguous [11]. For the isoparametric element, uniqueness of coordinates ensures compatibility. Continuity is assured when adjacent elements are given the same sets of coordinates at common nodes.

Since the element shape functions are in terms of local coordinates, an element of volume, dxdydz, must be transformed into an element of volume expressed in local coordinates. This is achieved through the use of the Jacobian matrix defined below. Using the chain rule, the relationship between ξ,η,ζ and a corresponding set of cartesian coordinates x,y,z is

$$\begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi}, & \frac{\partial y}{\partial \xi}, & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta}, & \frac{\partial y}{\partial \eta}, & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta}, & \frac{\partial y}{\partial \zeta}, & \frac{\partial z}{\partial \zeta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} \tag{48}$$

The Jacobian matrix [J] is defined as

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi}, & \frac{\partial y}{\partial \xi}, & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta}, & \frac{\partial y}{\partial \eta}, & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta}, & \frac{\partial y}{\partial \zeta}, & \frac{\partial z}{\partial \zeta} \end{bmatrix} \tag{49}$$

From equation (47), the Jacobian matrix becomes

$$[J] = \begin{bmatrix} \sum_i \frac{\partial N_i}{\partial \xi} x_i, & \sum_i \frac{\partial N_i}{\partial \xi} y_i, & \sum_i \frac{\partial N_i}{\partial \xi} z_i \\ \sum_i \frac{\partial N_i}{\partial \eta} x_i, & \sum_i \frac{\partial N_i}{\partial \eta} y_i, & \sum_i \frac{\partial N_i}{\partial \eta} z_i \\ \sum_i \frac{\partial N_i}{\partial \zeta} x_i, & \sum_i \frac{\partial N_i}{\partial \zeta} y_i, & \sum_i \frac{\partial N_i}{\partial \zeta} z_i \end{bmatrix} \tag{50}$$

The determinant of the Jacobian is used to transform the volume of element in cartesian coordinates to local coordinates. For a volume of element [11],

$$dxdydz = \det [J] \, d\xi d\eta d\zeta \tag{51}$$

Note that the determinant of the Jacobian is a variable for elements of curved geometry. Only in the case of straight-sided elements is the determinant of the Jacobian a constant.

In the plane of the triangle, the area coordinates (L_1, L_2,L_3) number one more than the cartesian coordinates (x,y). Thus, L_3 is defined as a dependent variable. This establishes the origin of the $\xi\eta$ coordinate system at corner point 3, as

illustrated in figure 11. Recall that the $\xi\eta$ axes need not be orthogonal. As such

$$\xi = L_1 \tag{52a}$$

$$\eta = L_2 \tag{52b}$$

Using equation (42),

$$L_3 = 1 - \xi - \eta \tag{52c}$$

Applying the chain rule yields

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial L_1}\frac{\partial L_1}{\partial \xi} + \frac{\partial N_i}{\partial L_2}\frac{\partial L_2}{\partial \xi} + \frac{\partial N_i}{\partial L_3}\frac{\partial L_3}{\partial \xi} \tag{53a}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial L_1}\frac{\partial L_1}{\partial \eta} + \frac{\partial N_i}{\partial L_2}\frac{\partial L_2}{\partial \eta} + \frac{\partial N_i}{\partial L_3}\frac{\partial L_3}{\partial \eta} \tag{53b}$$

Using equations (52a), (52b), and (52c) in equations (53a) and (53b) gives

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \tag{54a}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \tag{54b}$$

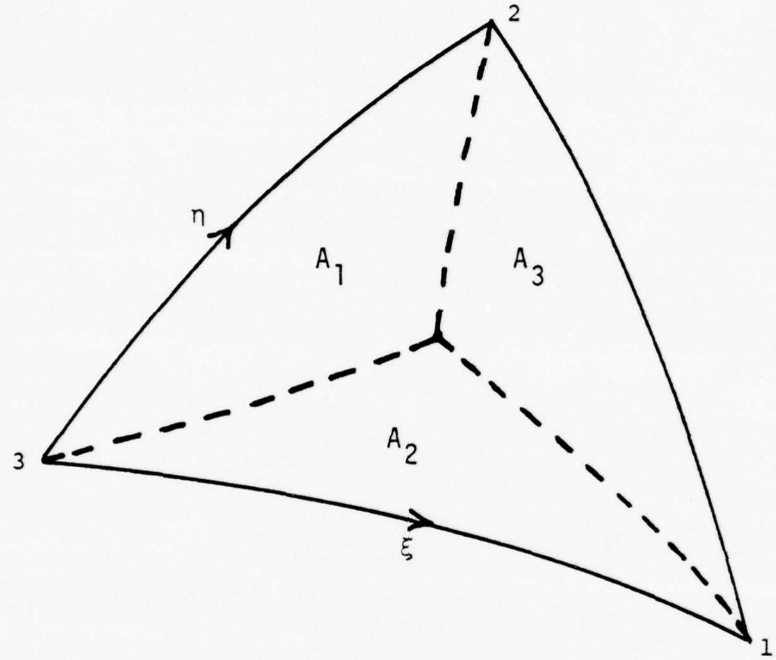


Figure 11. $\eta\xi$ coordinates in a triangle

Now the Jacobian matrix can be evaluated using the shape functions $N_i = N_i(L_1,L_2,L_3,\zeta)$. Inserting equations (54a) and (54b) in equation (50) produces

$$[J(L_1L_2,L_3,\zeta)] = \begin{bmatrix} \sum_i(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3})x_i \ , & \sum_i(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3})y_i \ , & \sum_i(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3})z_i \\ \sum_i(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3})x_i \ , & \sum_i(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3})y_i \ , & \sum_i(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3})z_i \\ \sum_i(\frac{\partial N_i}{\partial \zeta})x_i \ , & \sum_i(\frac{\partial N_i}{\partial \zeta})y_i \ , & \sum_i(\frac{\partial N_i}{\partial \zeta})z_i \end{bmatrix} \tag{55}$$

To summarize, *suppose* it is required to transform the integral

$$I = \int_{Vol} F'(L_1,L_2,L_3,\zeta)\ dxdydz \tag{56}$$

to an integral entirely in terms of local coordinates. The determinant of equation (55) is then utilized to give

$$I = \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-L_1} F'(L_1,L_2,L_3,\zeta)\det[J(L_1,L_2,L_3,\zeta)]dL_1dL_2d\zeta \tag{57}$$

Equation (56) is now in a form suitable for numerical integration.

The development of coordinate transformation to this point has been under the general assumption that all the sides of the element are curved. In this work, the only

curved side is in the plane of the triangle. The sides of the element along the prism axis are straight. As such, a modified Jacobian matrix can be employed, thereby reducing the number of calculations to be performed. This modified Jacobian is the 2x2 matrix defined by

$$[J^*] = \begin{bmatrix} \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) x_i \ , & \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) y_i \\ \sum_i (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) x_i \ , & \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) y_i \end{bmatrix} \quad (58)$$

Along the prism axis or ζ direction, it can be shown that

$$dz = \frac{h}{2} d\zeta \quad (59)$$

where h = height of the element. The volume relationship is then given by

$$dxdydz = \frac{h}{2} \det[J^*] \ dL_1 dL_2 \zeta \quad (60)$$

The integral of equation (56) then assumes the form of

$$I = \frac{h}{2} \int_{-1}^{1} \int_{0}^{1} \int_{0}^{1-L_1} F'(L_1,L_2,L_3,\zeta) \det[J^*(L_1,L_2,L_3,\zeta)] dL_1, dL_2, d\zeta \quad (61)$$

Equation (61) is the basis of numerical integration applied in this work.

F. CONSTRUCTION OF ELEMENT MATRICES

The system matrix operators can be constructed through the use of the global basis functions G_J or through the

application of the element shape functions. Although the global basis functions were used to demonstrate the method of Galerkin, the construction of the system matrices in this work was achieved through element considerations. The solution of the unknown variable ψ within the element domain was approximated by

$$\tilde{\psi}^e = \sum_{i=1} N_i \; \psi_i^e(t) \;\; , \quad i=1,2,\ldots,n^e \qquad (62)$$

where n^e is the number of element nodal points, N_i are the element shape functions, and $\psi_i^e(t)$ are the time-dependent nodal magnitudes of $\tilde{\psi}^e$. The element contribution to the system matrix operators is defined by Galerkin's orthogonality condition expressed by

$$\int_{Vol} N_j \; R^e \; dVol = 0 \;, \; j=1,2,\ldots,n^e \qquad (63)$$

where R^e is defined by replacing $\tilde{\psi}$ with $\tilde{\psi}^e$ in equations (37) and (38), and the integration is over the element volume. Using equation (62), the element contribution is portrayed for the core by

$$[\int_{Vol} N_j N_i dVol]\{\dot{\psi}_i{}^e(t)\} - [\int_{Vol} N_j \nabla^2 N_i dVol]\{(c1\cdot\psi^e)_i\} + [\int_{Vol} N_j N_i dVol]\{(c2\cdot\psi^e)_i\}$$
$$+ [\int_{Vol} N_j N_i \ell n(1 + \frac{K}{\gamma T_o} \sum_k N_k \; \psi_k{}^e(t)) dVol] \; \{(c4\cdot\psi^e)_i\}$$
$$+ [\int_{Vol} N_j N_i dVol] \; \{\int_o^t e^{-\bar{\lambda}(t-t')} \; (c5\cdot\psi(t'))_i dt'\} = 0 \qquad (64)$$

where $i,j,k = 1,2,\ldots,15$, and c1, c2, etc., are constants at node i. The bracketed expressions represent square matrices of size 15x15, and the braced expressions represent column vectors of size 15x1. For the reflector, the last two terms of equation (64) are zero. Before any operation can be performed on equation (64), the nonlinear terms (last two terms) must be "linearized" and the term with the ∇^2 operator must be integrated by parts.

The nonlinear feedback term, $\ell n[1 + \frac{K}{\gamma T_o} \sum_{k=1}^{15} N_k \psi_k{}^e(t)]$, was linearized by using predicted values of the unknown function at time t. These predicted values come from the time integration scheme by Franke [7]. Basically, the integration scheme utilizes a predictor-corrector method which predicts values of the unknown function at the next time by using the derivatives of the function. Adopting the predicted values $\psi_k{}^P$ enabled integration over space. The term in equation (64) involving the nonlinear feedback can be written as

$$[\int_{Vol} N_j N_i \ell n(1 + \frac{K}{\gamma T_o}(N_1\psi_1{}^P + N_2\psi_2{}^P + \ldots + N_{15}\psi_{15}{}^P))dVol] \{(c4\cdot\psi^e)_i\}$$

The last tern of equation (64) describes the delayed neutron contribution. In general,

$$e^{-\bar{\lambda}t_n} \int_0^{t_n} e^{\bar{\lambda}t'} \psi_i{}^e(t')dt' = e^{-\bar{\lambda}(t_n - t_{n-1})} [e^{-\bar{\lambda}t_{n-1}} \int_0^{t_{n-1}} e^{\bar{\lambda}t'} \psi_i{}^e(t')dt'] + e^{-\bar{\lambda}t_n} \int_{t_{n-1}}^{t_n} e^{\bar{\lambda}t'} \psi_i{}^e(t')dt' \quad (65)$$

where n is number of time steps, t_n is the current time,

and t_{n-1} is the previous time. To approximate the integrals in equation (65), the predicted values ψ_i^P were employed in a simple trapezoidal rule. The trapezoidal rule was believed to be sufficient since very small time steps were utilized in this work. For example, at time t_2 ,

$$(SUM_i) = (\text{previous } SUM_i)e^{-\bar{\lambda}H} + \frac{H}{2}(e^{\bar{\lambda}H}\psi_i^e(t_1) + \psi_i^P)$$

where

$$H = \text{current time step} = t_2 - t_1$$

$$(SUM_i) = e^{-\bar{\lambda}t_2} \int_0^{t_2} e^{\bar{\lambda}t'} \psi_i^e(t')dt'$$

$$(\text{previous } SUM_i) = e^{-\bar{\lambda}t_1} \int_0^{t_1} e^{\bar{\lambda}t'} \psi_i^e(t')dt'$$

The previous sum is formed by accumulating the trapezoidal integration of each time step. In general then, for time t_n

$$(SUM_i) = (\text{previous } SUM_i)e^{-\bar{\lambda}H} + \frac{H}{2}(e^{\bar{\lambda}H}\psi_i^e(t_{n-1}) + \psi_i^P) \quad (66)$$

The delayed neutron term in equation (64) can be expressed as

$$[\int_{Vol} N_j N_i dVol] \{(c5 \cdot SUM)_i\}$$

In order to bring equation (64) to final form, the ∇^2 operator was integrated by parts. Since the flux at the surface of the reactor is zero, integration by parts yields

$$\int_{Vol} N_j \nabla^2 N_i dxdydz = -\int_{Vol} \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\right)dxdydz \quad (67)$$

In vector notation, equation (67) is expressed as

$$\int_{Vol} N_j \nabla^2 N_i dxdydz = -\int_{Vol} \left\langle \frac{\partial N_i}{\partial x}, \frac{\partial N_i}{\partial y}, \frac{\partial N_i}{\partial z} \right\rangle \begin{Bmatrix} \frac{\partial N_j}{\partial x} \\ \frac{\partial N_j}{\partial y} \\ \frac{\partial N_j}{\partial z} \end{Bmatrix} dxdydz \quad (68)$$

Using the chain rule produces

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = \begin{bmatrix} \frac{\partial L_1}{\partial x}, & \frac{\partial L_2}{\partial x}, & 0 \\ \frac{\partial L_1}{\partial y}, & \frac{\partial L_2}{\partial y}, & 0 \\ 0, & 0, & \frac{\partial \zeta}{\partial z} \end{bmatrix} \begin{Bmatrix} \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}\right) \\ \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}\right) \\ \left(\frac{\partial N_i}{\partial \zeta}\right) \end{Bmatrix} \quad (69)$$

Letting the 3x3 matrix above be [B'], the ∇^2 term becomes

$$\int_{Vol} N_j \nabla^2 N_i dxdydz = -\int_{Vol} \left\langle \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}\right), \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}\right), \frac{\partial N_i}{\partial \zeta} \right\rangle [B']^T[B'] \begin{Bmatrix} \left(\frac{\partial N_j}{\partial L_1} - \frac{\partial N_j}{\partial L_3}\right) \\ \left(\frac{\partial N_j}{\partial L_2} - \frac{\partial N_j}{\partial L_3}\right) \\ \frac{\partial N_j}{\partial \zeta} \end{Bmatrix} dxdydz \quad (70)$$

where $[B']^T$ is the transpose of $[B']$. By applying the chain rule in equation (47) and using equation (59), it can be shown that for this work

$$[B'] = \begin{bmatrix} \dfrac{1}{(\frac{\partial N_1}{\partial L_1} x_1 + \ldots + \frac{\partial N_{15}}{\partial L_1} x_{15})}, & \dfrac{1}{(\frac{\partial N_1}{\partial L_2} x_1 + \ldots + \frac{\partial N_{15}}{\partial L_2} x_{15})}, & 0 \\ \dfrac{1}{(\frac{\partial N_1}{\partial L_1} y_1 + \ldots + \frac{\partial N_{15}}{\partial L_1} y_{15})}, & \dfrac{1}{(\frac{\partial N_1}{\partial L_2} y_1 + \ldots + \frac{\partial N_{15}}{\partial L_2} y_{15})}, & 0 \\ 0 \quad , & 0 \quad , & \dfrac{2}{h} \end{bmatrix} \tag{71}$$

$[B']^T$ can be derived from equation (71).

Note from equation (64) that there are three basic element matrices which are defined as follows after applying equation (60):

$$[G_{ji}] = \frac{h}{2} \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-L_1} N_j N_i \det[J^*] dL_1 dL_2 d\zeta \tag{72}$$

$$[GG_{ji}] = \frac{h}{2} \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-L_1} \left\langle (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}), (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}), (\frac{\partial N_i}{\partial \zeta}) \right\rangle [B']^T$$

$$[B'] \begin{Bmatrix} (\frac{\partial N_j}{\partial L_1} - \frac{\partial N_j}{\partial L_3}) \\ (\frac{\partial N_j}{\partial L_2} - \frac{\partial N_j}{\partial L_3}) \\ \frac{\partial N_j}{\partial \zeta} \end{Bmatrix} \det[J^*] dL_1 dL_2 d\zeta \tag{73}$$

$$[GGG_{ji}] = \frac{h}{2} \int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-L_1} N_j N_i \ell n(1 + \frac{K}{\gamma T_o}(N_1 \psi_1{}^P + \ldots + N_{15} \psi_{15}^P)) \det[J^*] dL_1 dL_2 d\zeta$$

(74)

Element matrices $[G_{ji}]$ and $[GG_{ji}]$ are independent of time. However, $[GGG_{ji}]$ is time dependent due to the utilization of the predicted values ψ_i^P which changes with time. In terms of these three basic element matrices, equation (64) is

$$[G_{ji}]\{\dot{\psi}_i^e\} + [GG_{ji}]\{c1\cdot\psi^e)_i\} + [G_{ji}]\{(c2\cdot\psi^e)_i\}$$

$$+ [GGG_{ji}]\ \{(c4\cdot\psi^e)_i\} + [G_{ji}]\ \{(c5\cdot SUM)_i\} = 0 \qquad (75)$$

The last two terms of equation (75) are zero for the reflector.

G. CONSTRUCTION OF THE SYSTEM MATRICES

The 15x15 coefficient element matrices were calculated according to equations (72), (73), and (74); and the results were collected element by element into the corresponding system coefficient matrices. The system coefficient matrix [BIGG] is developed from $[G_{ji}]$, [BIGGG] from $[GG_{ji}]$, and [BIGH] from $[GGG_{ji}]$. [BIGG] and [BIGGG] are independent of time and can be constructed once and for all from geometry considerations. [BIGH] is dependent on both geometry and time due to the time dependence of the predicted flux utilized in the feedback term. Thus, [BIGH] is recalculated at each time increment.

Non-zero contributions to a global nodal point I come only from adjacent elements sharing that same nodal point I. Thus, the system matrices are sparse and banded. The process of assembling contributions from element matrices

requires the identification of a local nodal point (i=1,2, ...,15) with a global nodal point (I=1,2,...NUMNP, where NUMNP is the total number of system nodes). This correspondence between element and global nodes is accomplished via the connectivity matrix.

The formal treatment of the field equations in terms of the system coefficient matrices is described by the equation

$$[BIGG]\ \{\dot{\psi}_I\} + [BIGGG]\ \{(c1\cdot\psi)_I\} + [BIGG]\ \{(c2\cdot\psi)_I\}$$
$$+ [BIGH]\ \{(c4\cdot\psi)_I\} + [BIGG]\ \{(c5\cdot SUM)_I\} = 0 \qquad (76)$$

where the system matrices are NUMNP x NUMNP and the column vectors are of length NUMNP x 1. However, the direct application of equation (76) requires a large amount of computer storage. To take advantage of the sparsity of the system matrices, an optimum compacting scheme described by Ref. 8 was employed.

The concept behind OCS is simply to store only the non-zero terms of a coefficient matrix. OCS requires two integer arrays, say JB and NAME, and a vector of non-zero coefficients of the square system matrix. For purposes of illustration, the square system matrix is called B, and the vector of non-zero coefficients of B is called BB. The i^{th} integer entry in the NUMNP x 1 JB vector is the number q_i. This number is defined by

$$q_i = 1 + \sum_{j=1}^{i=1} p_j \ , \ i=1,2,\ldots,NUMNP \qquad (77)$$

where p_j is the number of terms in the i^{th} equation. In

other words, p_j is the number of nodes that the i^{th} node "sees". JB is therefore a pointer vector of length NUMNP+1 whose i^{th} term locates the initial position in the BB vector of the contributing coefficients to the i^{th} equation. The NAME vector of length Mx1, where $M = \sum_{i=1}^{NUMNP} p_i$, consists of NUMNP successive vector blocks of variable length p_i, i=1,2,...,NUMNP. The p_i integer numbers in the i^{th} block of NAME list the p_i contributors to the i^{th} equation. The Mx1 BB vector contains the real non-zero coefficients of the NUMNPxNUMNP B matrix, arranged in the same contiguous block arrangement as the NAME vector. The j^{th} term in the i^{th} block or BB(JB(I) + J-1) is B(I,K), where K = NAME(JB(I)+J-1). To illustrate, consider the grid shown in figure 12. The two array vectors and the coefficient vector matrix of non-zero terms are:

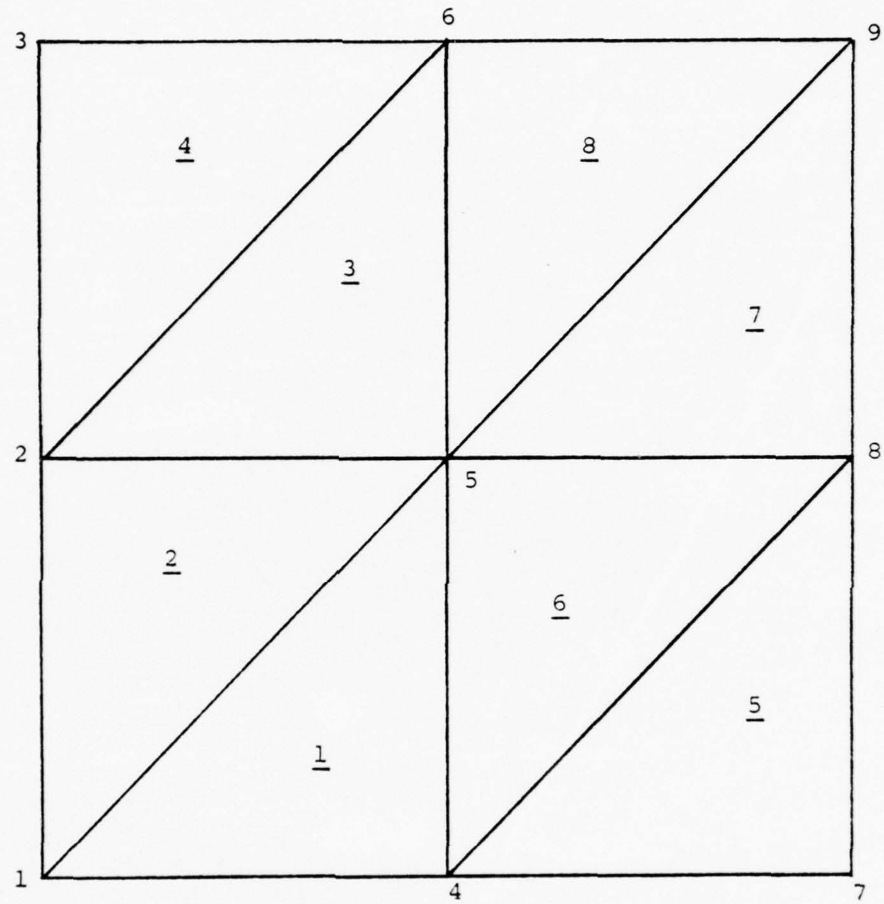
$$JB = \langle 1, 5, 10, 13, 18, 25, 30, 33, 38, 42 \rangle$$

$$NAME = \langle 1,2,4,5 | 2,3,1,6,5 | \text{---} | 9,8,6,5 \rangle$$

$$BB = \langle B_{11}, B_{12}, B_{14}, B_{15} | B_{22}, B_{23}, B_{21}, B_{26}, B_{25} | \text{---} | B_{99}, B_{98}, B_{96}, B_{95} \rangle$$

In this illustration, NUMNP = 9 and M = 41 [8].

In this work, a judicious method of numbering the system nodal points was adopted to further reduce computer storage requirements. Since the surface boundary nodes of the reactor represent zero neutron fluxes, the contributions of these nodes to interior or non-zero nodes can be discarded. Thus, only the interior nodal points need to be considered. These non-zero nodes were numbered first in the finite element mesh



Number - element number

Figure 12. Sample grid used for illustrating OCS

used so that in the OCS, the number of non-zero nodes (NNZ) replaces NUMNP.

The vectors of non-zero coefficients will be designated BIGG, BIGGG and BIGH since the square coefficient matrices described in equation (76) were not utilized. To illustrate the application of OCS in the system, the following sample program is given:

```
    DO  450  I=1, NNZ
    JBB = JB(I)
    JE = JB(I+1)-1
    DY(I) = 0.0
    DO  500  J=JBB, JE
    LL = NAME(J)
    DY(I) = DY(I) + BIGG(J)*ψ̇(LL) + BIGGG(J)*c1(LL)*
    ψ(LL) + BIGG(J)*c2(LL)*ψ(LL) + BIGH(J)*c4(LL)*
    ψ(LL) + BIGG(J)*c5(LL)*SUM(LL)                      (78)
500 CONTINUE
450 CONTINUE
```

where

LL = nodal point "seen" by node I

JE-JBB = total number of nodal points that node I "sees"

DY(I) = summation of all contributions to node I and should sum to zero as stated by the field equation

The assumption of a homogeneous reflector was relaxed in the application of equation (78). Interface nodes were assigned properties of the core. Therefore, if LL is a reflector

node not on the core reflector interface, the last two terms of equation (78) are nonexistent.

IV. NUMERICAL INTEGRATION

A. LINE AND AREA INTEGRATION

The solution of the element matrix equations was achieved through numerical integration since an exact closed form solution cannot be established. The volume integration was accomplished by using a line integration in the ζ-direction and an area integration in the plane of the triangle. The line integral is described by the Gaussian quadrature formula [12]

$$\int_{-1}^{1} f(\zeta)d\zeta \approx \sum_{k=1}^{n} H_k f(a_k) \tag{79}$$

where n is the number of Gauss integration points

H_k = weighting coefficients

$f(a_k)$ = the function $f(\zeta)$ evaluated at Gauss point a_R

Table III lists a_k, H_k and n [11].

The area integration was achieved by the equation

$$\int_0^1 \int_0^{1-L_1} f(L_1,L_2,L_3)dL_1 dL_2 \approx \sum_{m=1}^{\bar{m}} w_m f(L_1{}^m,L_2{}^m,L_3{}^m) \tag{80}$$

where $\bar{m}$ is the number of area integration points and w_m are the weights. The numerical integration points for the area integration are given in Table IV which was extracted

TABLE III

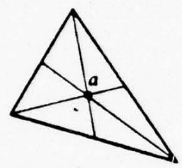
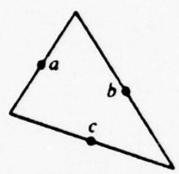
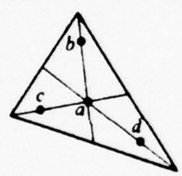
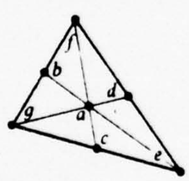
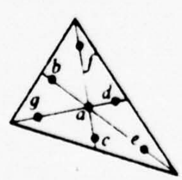
Abscissae and Weight Coefficients of the Gaussian Quadrature Formula

$$\int_{-1}^{1} f(\zeta)\,d\zeta = \sum_{k=1}^{n} H_k f(a_k)$$

$\pm a$	H
$n = 2$	
0·57735 02691 89626	1·00000 00000 00000
$n = 3$	
0·77459 66692 41483	0·55555 55555 55556
0·00000 00000 00000	0·88888 88888 88889
$n = 4$	
0·86113 63115 94053	0·34785 48451 37454
0·33998 10435 84856	0·65214 51548 62546
$n = 5$	
0·90617 98459 38664	0·23692 68850 56189
0·53846 93101 05683	0·47862 86704 99366
0·00000 00000 00000	0·56888 88888 88889
$n = 6$	
0·93246 95142 03152	0·17132 44923 79170
0·66120 93864 66265	0·36076 15730 48139
0·23861 91860 83197	0·46791 39345 72691
$n = 7$	
0·94910 79123 42759	0·12948 49661 68870
0·74153 11855 99394	0·27970 53914 89277
0·40584 51513 77397	0·38183 00505 05119
0·00000 00000 00000	0·41795 91836 73469
$n = 8$	
0·96028 98564 97536	0·10122 85362 90376
0·79666 64774 13627	0·22238 10344 53374
0·52553 24099 16329	0·31370 66458 77887
0·18343 46424 95650	0·36268 37833 78362
$n = 9$	
0·96816 02395 07626	0·08127 43883 61574
0·83603 11073 26636	0·18064 81606 94857
0·61337 14327 00590	0·26061 06964 02935
0·32425 34234 03809	0·31234 70770 40003
0·00000 00000 00000	0·33023 93550 01260
$n = 10$	
0·97390 65285 17172	0·06667 13443 08688
0·86506 33666 88985	0·14945 13491 50581
0·67940 95682 99024	0·21908 63625 15982
0·43339 53941 29247	0·26926 67193 09996
0·14887 43389 81631	0·29552 42247 14753

TABLE IV

Numerical Formulas for Triangles

Order	Fig.	Error	Points	Triangular Co-ordinates	Weights $2W_k$
Linear		$R = 0(h^2)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	1
Quadratic		$R = 0(h^3)$	a	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{1}{3}$
			b	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{3}$
			c	$\frac{1}{2}, 0, \frac{1}{2}$	$\frac{1}{3}$
Cubic		$R = 0(h^4)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$-\frac{27}{48}$
			b	$\frac{11}{15}, \frac{2}{15}, \frac{2}{15}$	$\frac{25}{48}$
			c	$\frac{2}{15}, \frac{11}{15}, \frac{2}{15}$	
			d	$\frac{2}{15}, \frac{2}{15}, \frac{11}{15}$	
Cubic		$R = 0(h^4)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$\frac{27}{60}$
			b	$\frac{1}{2}, \frac{1}{2}, 0$	
			c	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{8}{60}$ (b, c, d)
			d	$\frac{1}{2}, 0, \frac{1}{2}$	
			e	$1, 0, 0$	
			f	$0, 1, 0$	$\frac{3}{60}$ (e, f, g)
			g	$0, 0, 1$	
Quintic		$R = 0(h^6)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	0·225
			b	$\alpha_1, \beta_1, \beta_1$	
			c	$\beta_1, \alpha_1, \beta_1$	0·13239415
			d	$\beta_1, \beta_1, \alpha_1$	
			e	$\alpha_2, \beta_2, \beta_2$	
			f	$\beta_2, \alpha_2, \beta_2$	0·12593918
			g	$\beta_2, \beta_2, \alpha_2$	

with
$\alpha_1 = 0{\cdot}05971587$
$\beta_1 = 0{\cdot}47014206$
$\alpha_2 = 0{\cdot}79742699$
$\beta_2 = 0{\cdot}10128651$

from Ref. 11. The volume integration of the function $f(L_1, L_2, L_3, \zeta)$ using numerical integration is therefore

$$\int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-L_1} f(L_1, L_2, L_3, \zeta)\, dL_1\, dL_2\, d\zeta \approx$$

$$\sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m f(L_1{}^m, L_2{}^m, L_3{}^m, \zeta^k) \qquad (81)$$

where $w_k = H_k$. In the manner of equation (81), the element matrices become

$$[G_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m\ N_j(L_1{}^m, L_2{}^m, L_3{}^m, \zeta^k) N_i \det[J^*] \qquad (82)$$

$$[GG_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m\ F\,(L_1{}^m, L_2{}^m, L_3{}^m, \zeta^k)\ \det[J^*] \qquad (83)$$

$$[GGG_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m\ T\,(L_1{}^m, L_2{}^m, L_3{}^m, \zeta^k)\ \det[J^*] \qquad (84)$$

where F and T are easily derived from equations (73) and (74), respectively. Note that N_i and $\det[J^*]$ are also evaluated at each integration point. They are not shown as such merely for the sake of convenience in writing the equations.

B. *NUMBER OF INTEGRATION POINTS*

It is difficult to estimate the number of integration points required for good accuracy due to the complexity of the functions involved. The basic rule that the best number of integration points is found by trial and experience was adopted. For the $[G_{ji}]$ element matrix, the numbers involved

can be approximated. This was done by letting the determinant of the Jacobian equal to twice the area of the curved triangle (checking det[J*] at each integration point showed that this assumption was not too unreasonable). With det[J*] outside the integration process, $[G_{ji}]$ can be solved in closed form by integrating out ζ from -1 to +1 and then applying the closed form equation [12]

$$\int_{Area} L_1^{m_1} L_2^{m_2} L_3^{m_3} \, d(Area) = 2A \frac{m_1! m_2! m_3!}{(m_1+m_2+m_3+2)!} \tag{85}$$

where m_1, m_2, m_3 are positive integer exponents and A is the area of the triangle.

Five test points within the 15x15 element matrix were selected, as listed in Table V. The values obtained from the application of equation (85) are also given in Table V. Three sets of area integration points, each with a different number of ζ Gauss points, were used. These three sets of area integration points, given in Table IV, are:

1. cubic order (4 points)
2. cubic order (7 points)
3. quintic order (7 points)

Using each of the three integration points above with different ζ Gauss points in equation (82) produced the results obtained in Table V. From these results, the quintic order area integration points were selected for the element matrix $[G_{ji}]$ with three ζ Gauss points in the prism axes. This set of integration points was also used for $[GGG_{ji}]$.

TABLE V.

Selection of Integration Points for $[G_{ji}]$

ζ points	Area points	G(1,1)	G(2,2)	G(1,9)	G(6,9)	G(9,9)
13	cubic (4 pts)	1415	5278	-2756	5072	8536
5	"	1817	5278	-3170	5072	10240
7	"	1817	5278	-3170	5072	10240
3	cubic (7 pts)	3248	8247	-3114	4960	10243
5	"	3249	8247	-3114	4960	10240
7	"	3249	8247	-3114	4960	10240
3	quintic (7 pts)	2464	6600	-3144	5020	10240
5	"	2464	6600	-3144	5020	10240
7	"	2464	6600	-3144	5020	10240
Approximated values		2500	6700	-3142	5026	10053

The numbers for the $[GG_{ji}]$ element matrix could not be approximated. The best that could be done was to obtain an idea of the order of magnitude of this element matrix. Towards this end, a linear triangular element was assumed. Using linear approximation, the order of magnitude was found to be about 10^3. Using the three different area integration points mentioned above with varying ζ Gauss points in equation (83) yielded the results given in Table VI. Further checks on the $[GG_{ji}]$ element matrix showed that the cubic order with four area integration points yielded the desired order of magnitude. Thus, the fourth order cubic with five ζ Gauss points was employed for the $[GG_{ji}]$ element matrix. A note should be mentioned here in regards to the vast difference in results obtained for $[GG_{ji}]$ using different area integration points. Most likely, it was due to the $[B']$ and $[B']^T$ matrices which required the inversion of $\frac{\partial x}{\partial L_1}$, $\frac{\partial x}{\partial L_2}$, etc. Or perhaps it was caused by the nature of the hybrid element used in this work. In any case, further investigation is warranted in this area.

TABLE VI

Selection of Integration Points for $[GG_{ji}]$

ζ points	Area points	GG(1,9)	GG(3,3)	GG(14,9)	GG(8,8)	GG(15,15)
3	cubic (4 pts)	18.4	170.1	-165	165.9	106.1
5	"	22.0	177.6	-186	195.9	106.1
7	"	22.0	177.6	-186	195.9	106.1
3	cubic (7 pts)	-1.03×10^4	2.11×10^7	1.05×10^7	8.42×10^7	2056
5	"	-1.03×10^4	2.11×10^7	1.05×10^7	8.42×10^7	2056
7	"	-1.03×10^4	2.11×10^7	1.05×10^7	8.41×10^7	2056
3	quintic (7 pts)	-52.6	1.27×10^4	-6.64×10^4	6.84×10^4	1.51×10^5
5	"	-52.6	1.27×10^4	-6.64×10^4	6.84×10^4	1.51×10^5
7	"	-52.6	1.27×10^4	-6.64×10^4	6.84×10^4	1.51×10^5

V. TEST PROBLEMS AND RESULTS

The reactor was subjected to uniform and local perturbations in the form of a ramp input described by

$$\Sigma_f = \Sigma_f^* + \alpha t \qquad (86)$$

where α is the change in Σ_f per unit time and Σ_f^* is the critical fission cross-section. Σ_f^* must first be obtained before the perturbations can be applied. This was accomplished by trial and error until a stationary solution was reached. For mesh I, Σ_f^* was found to be 0.0057360 per cm. No attempt was made to find Σ_f^* for mesh II due to time limitations. As such, the test problems outlined below were applied to mesh I. Future work is planned to apply the test problems on mesh II.

The following perturbations were applied:

a) Uniform perturbation of 10 dollar of reactivity per second:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha t \,, \qquad \text{in the core}$$

where $\alpha = 0.005893/\text{cm-sec}$

b) Local perturbation at the core center of 100 dollar of reactivity per second:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha t \delta(\bar{r}_0) \,, \quad \text{in the core}$$

where $\bar{r}_0$ is (0,0,0) and α = 0.015407/cm-sec

c) Local, off-center perturbation:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha_i t \delta(\bar{r}_1) \ , \quad i = 1,2,3, \text{ in the core}$$

where

$$\bar{r}_1 = (0,60,40)$$

$$\alpha_1 = 0.015407/\text{cm-sec} = 100 \text{ dollar per second}$$

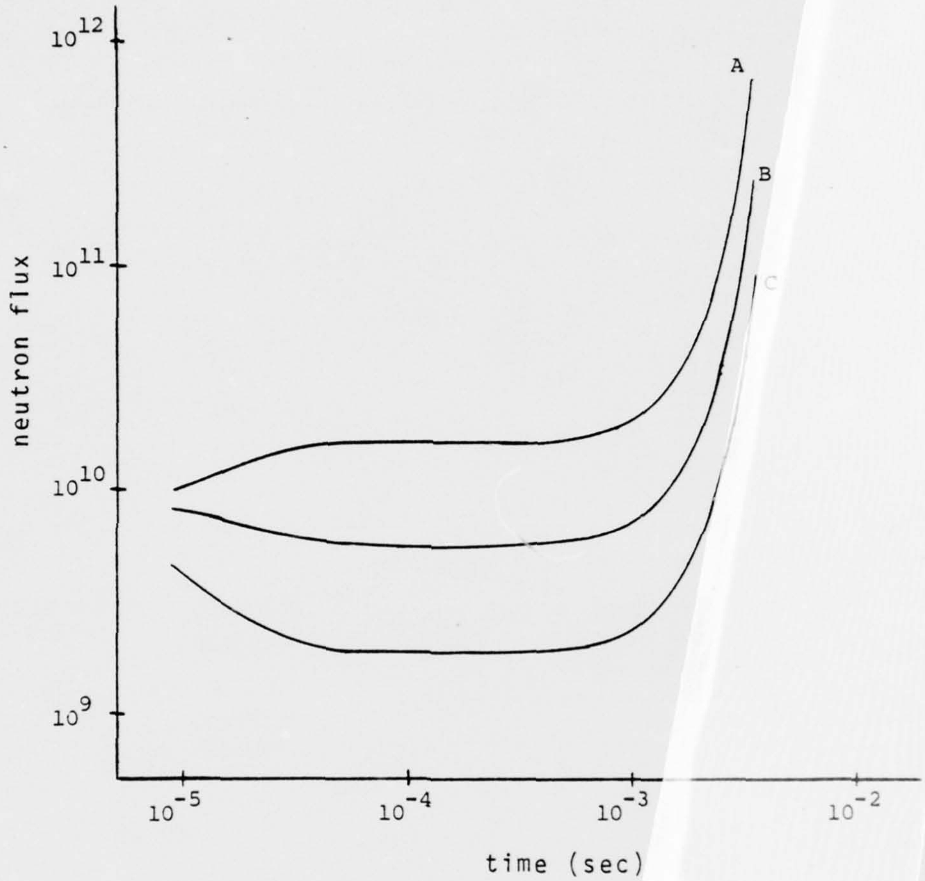
$$\alpha_2 = 0.008123/\text{cm-sec} = 50 \text{ dollar per second}$$

$$\alpha_3 = 0.005894/\text{cm-sec} = 10 \text{ dollar per second}$$

Three test points, (0,0,0), (60,0,0), and (-60,0,80), were selected to trace the neutron time history. For cases a) and b), the neutron flux was plotted at each test point during transience. This is shown in figures 13 and 14. Case c) involved three ramp inputs and were conducted for both the linear and nonlinear reactor equations. The linear and nonlinear responses were compared at each test point for each ramp input and are illustrated in figures 15 through 23. The radial and axial flux distributions at time t = 0.0123 second were plotted for the steady state and the 100 dollar perturbation case. These are embodied in figures 24 and 25. Finally, a neutron flux early time history between mesh I and mesh II was plotted to check the effect of using a finer element mesh. The result is portrayed in figure 26.

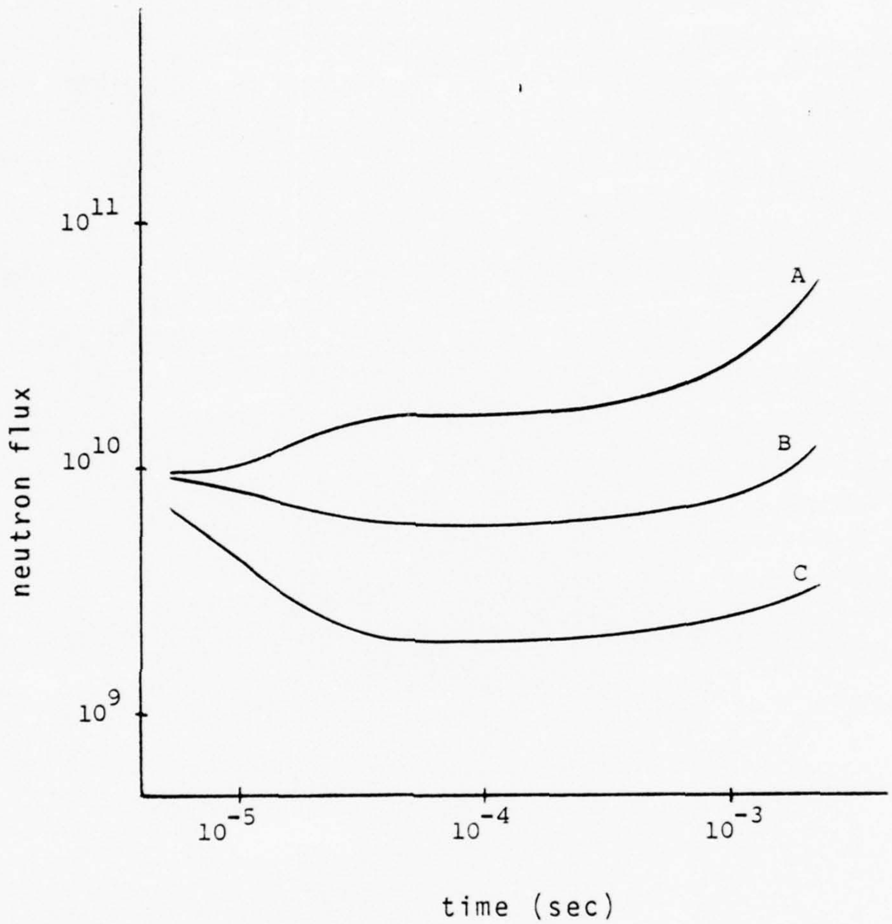
Figures 13 and 14 revealed a clear space dependence of the neutron flux during transience as expected. Figures 15 through 23 demonstrated the effects of temperature feedback

only the interior nodal points need to be considered. These non-zero nodes were numbered first in the finite element mesh



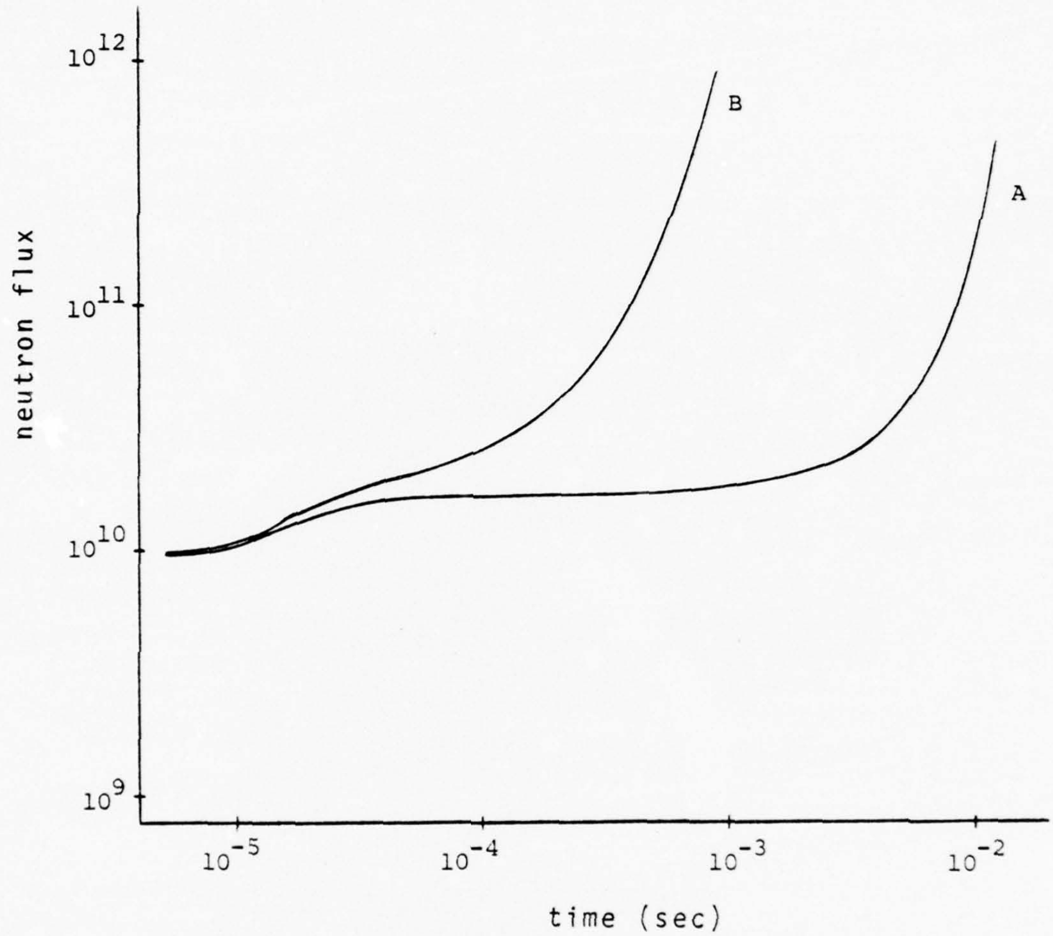
A - neutron flux at test point (0,0,0)
B - neutron flux at test point (60,0,0)
C - neutron flux at test point (-60,0,80)

Figure 13. Neutron flux transient history at three test points with a uniform perturbation of 10 dollar of reactivity per second.



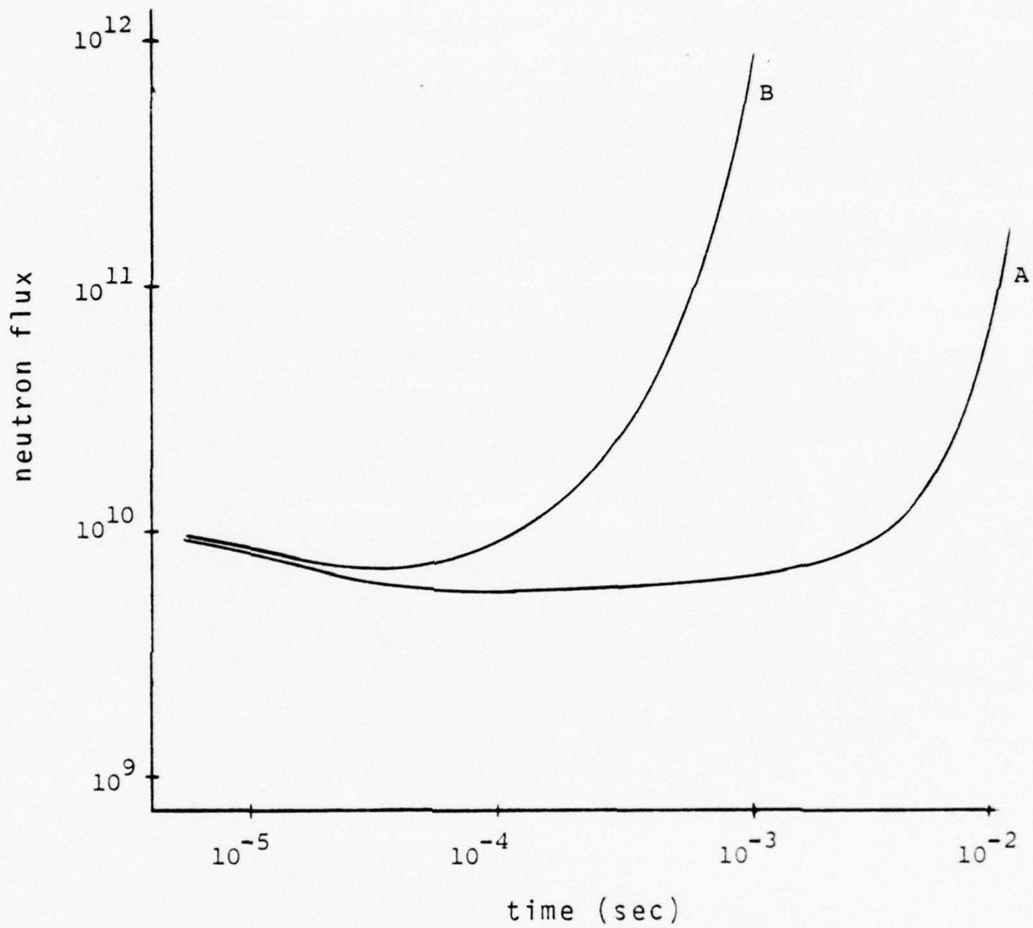
A - neutron flux at test point (0,0,0)
B - neutron flux at test point (60,0,0)
C - neutron flux at test point (-60,0,80)

Figure 14. Neutron flux transient history at various test points for a local central perturbation of 100 dollar/sec of reactivity.



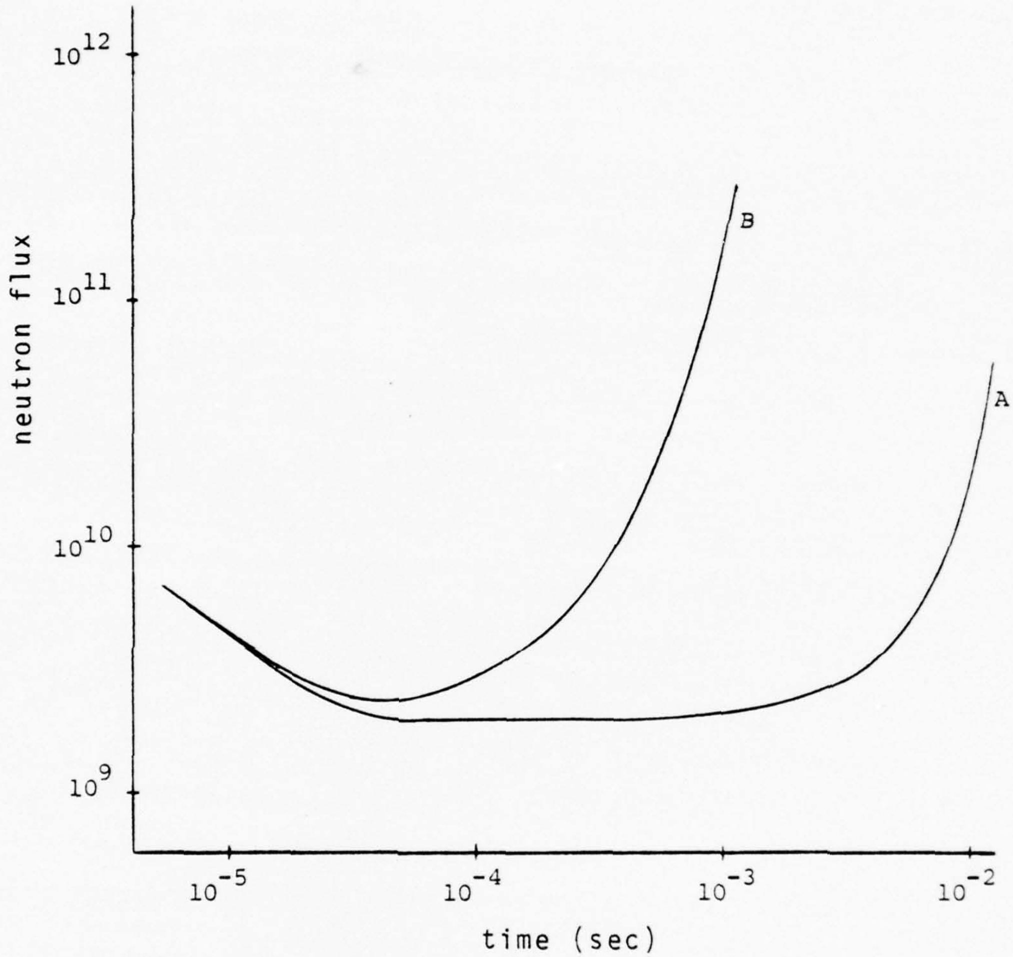
A - nonlinear neutron response
B - linear neutron response

Figure 15. Linear and nonlinear fluxes at (0,0,0) due to a local perturbation of 100 dollar/sec of reactivity at (0,60,40).



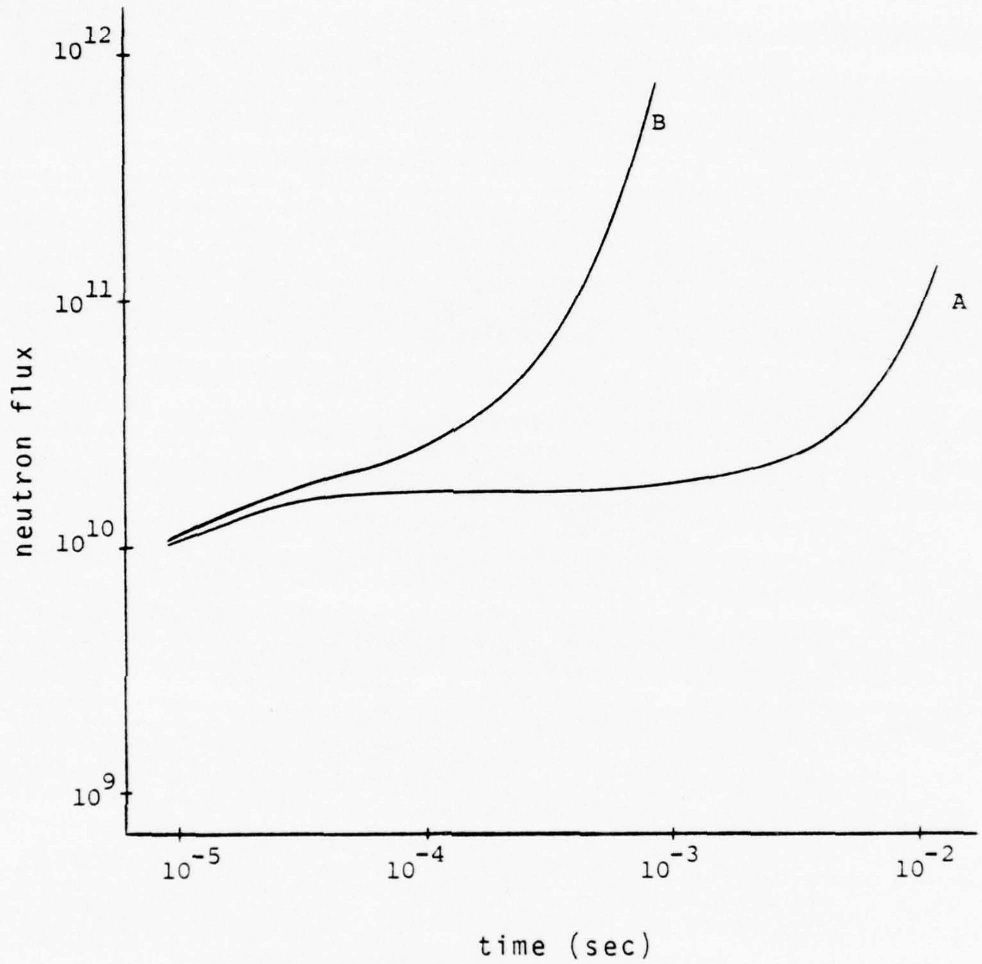
A - nonlinear neutron response
B - linear neutron response

Figure 16. Linear and nonlinear fluxes at (60,0,0) due to a local perturbation of 100 dollar/sec at (0,60,40).



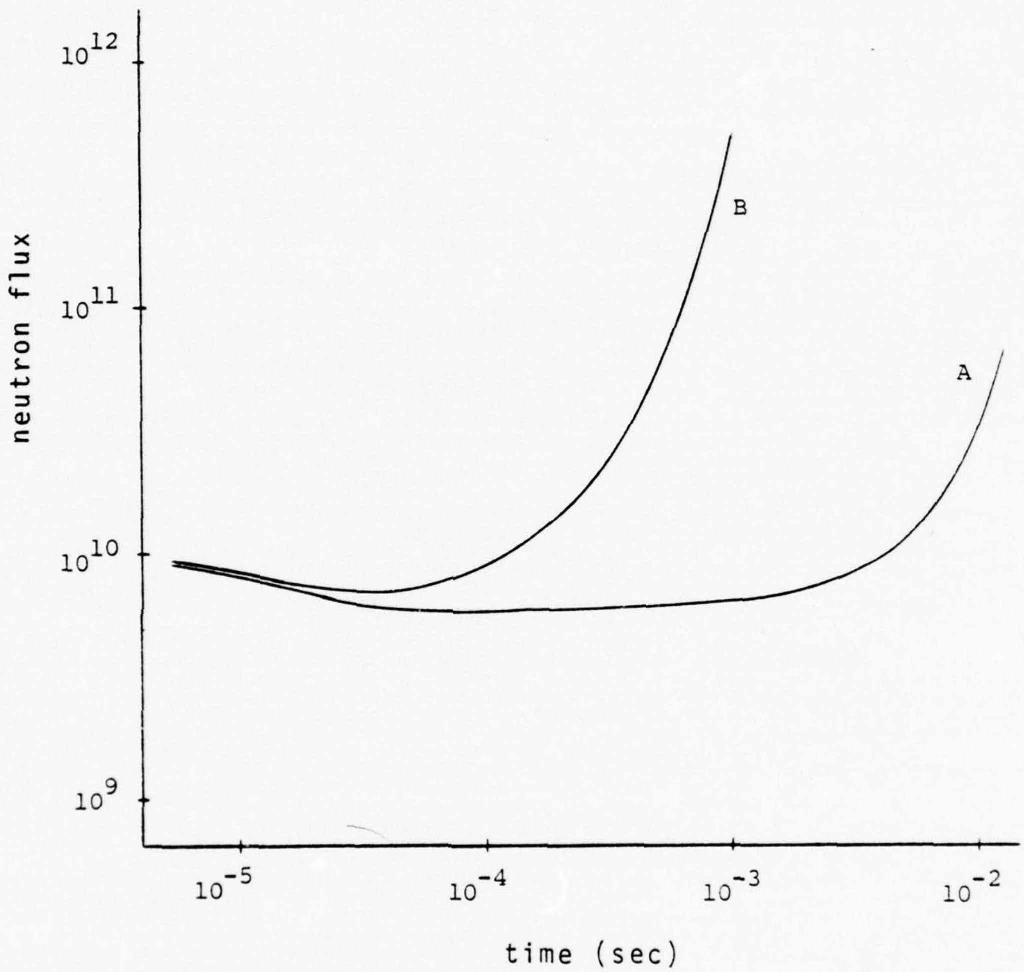
A - nonlinear neutron response
B - linear neutron response

Figure 17. Linear and nonlinear fluxes at (-60,0,80) due to a local perturbation of 100 dollar/sec at (0,60,40).



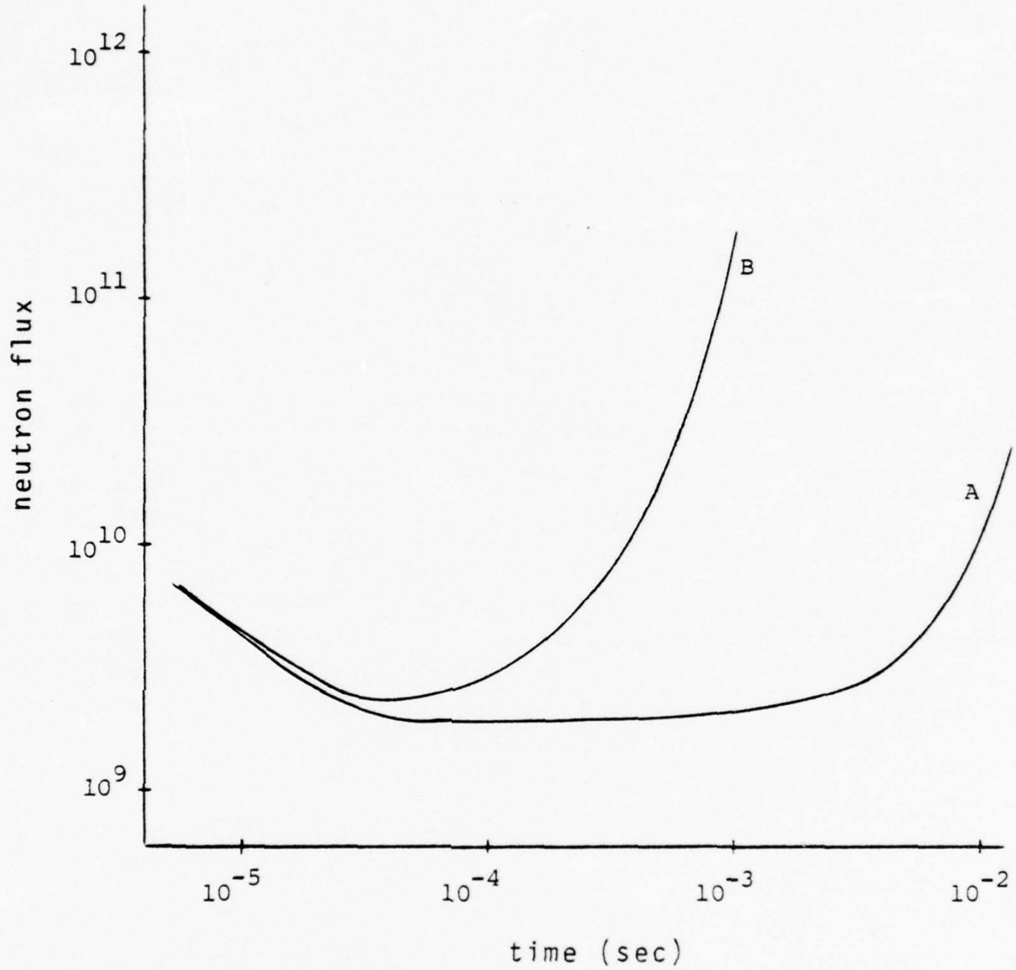
A - nonlinear neutron response
B - linear neutron response

Figure 18. Linear and nonlinear fluxes at (0,0,0) due to a 50 dollar/sec local perturbation at (0,60,40).



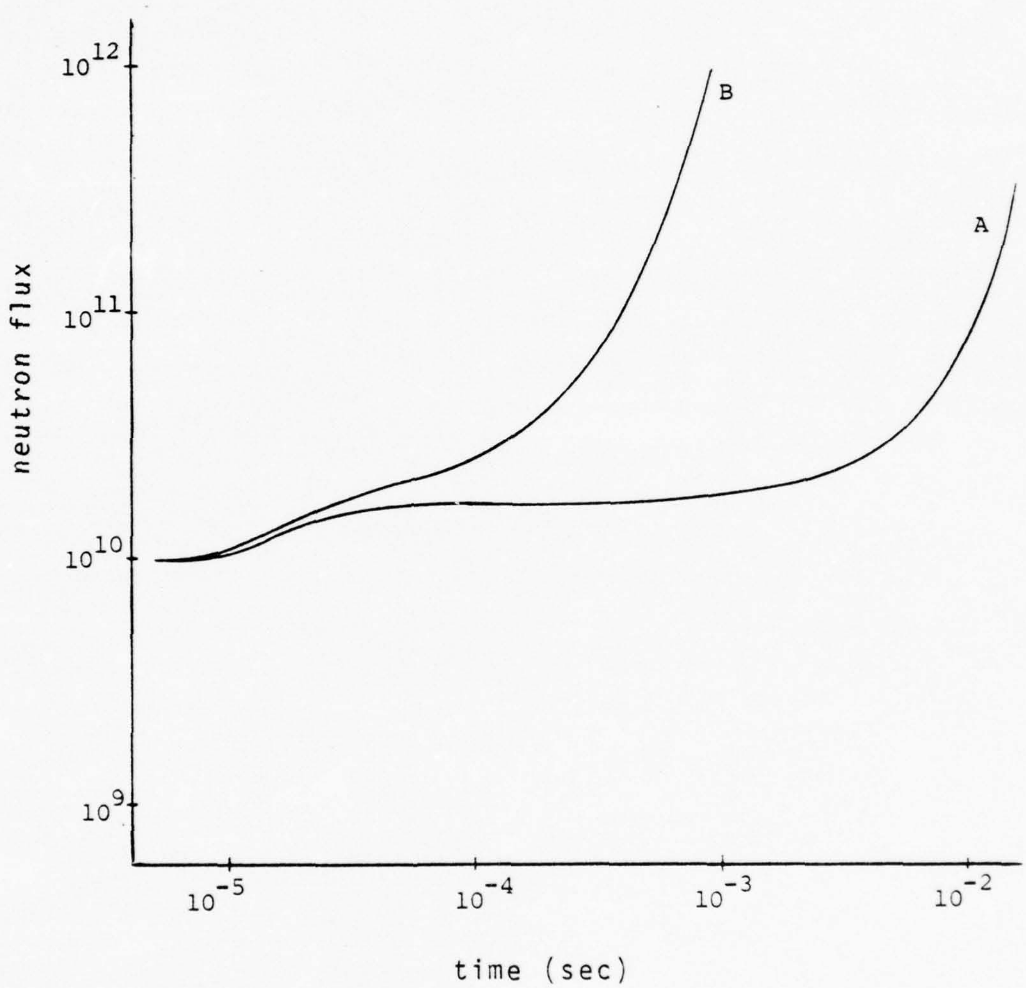
A - nonlinear neutron response
B - linear neutron response

Figure 19. Linear and nonlinear fluxes at (60,0,0) due to a 50 dollar/sec local perturbation at (0,60,40).



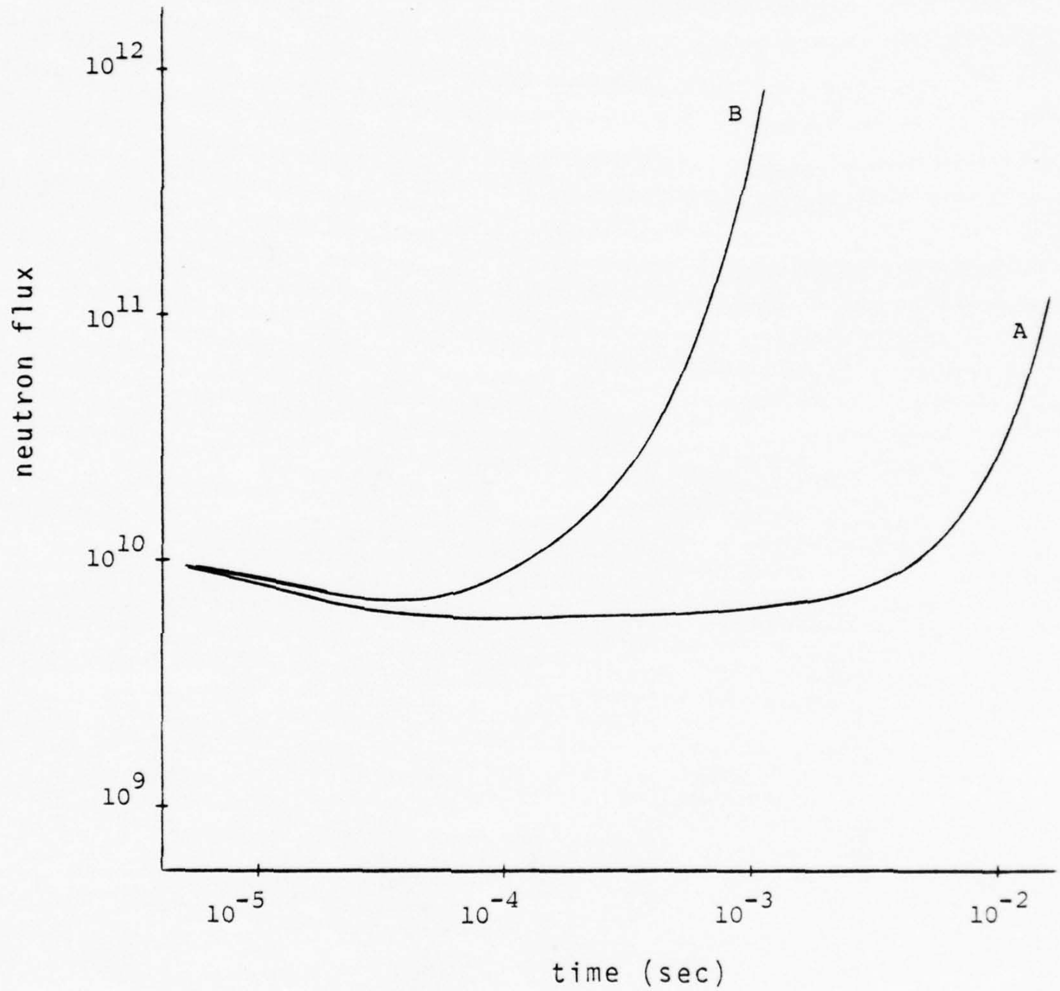
A - nonlinear neutron response
B - linear neutron response

Figure 20. Linear and nonlinear fluxes at (-60,0,80) due to a 50 dollar/sec local perturbation at (0,60,40).



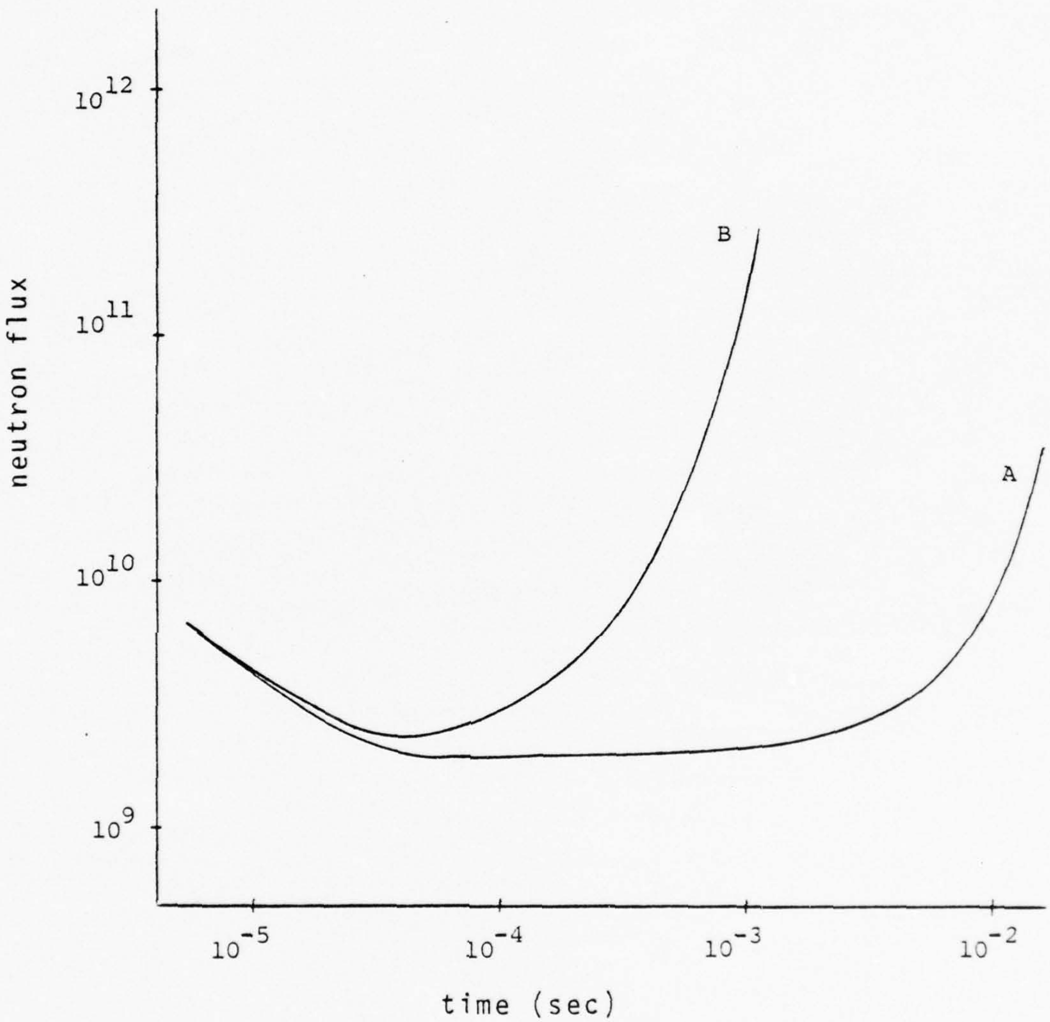
A - nonlinear neutron response
B - linear neutron response

Figure 21. Linear and nonlinear fluxes at (0,0,0) due to a 10 dollar/sec local perturbation at (0,60,40).



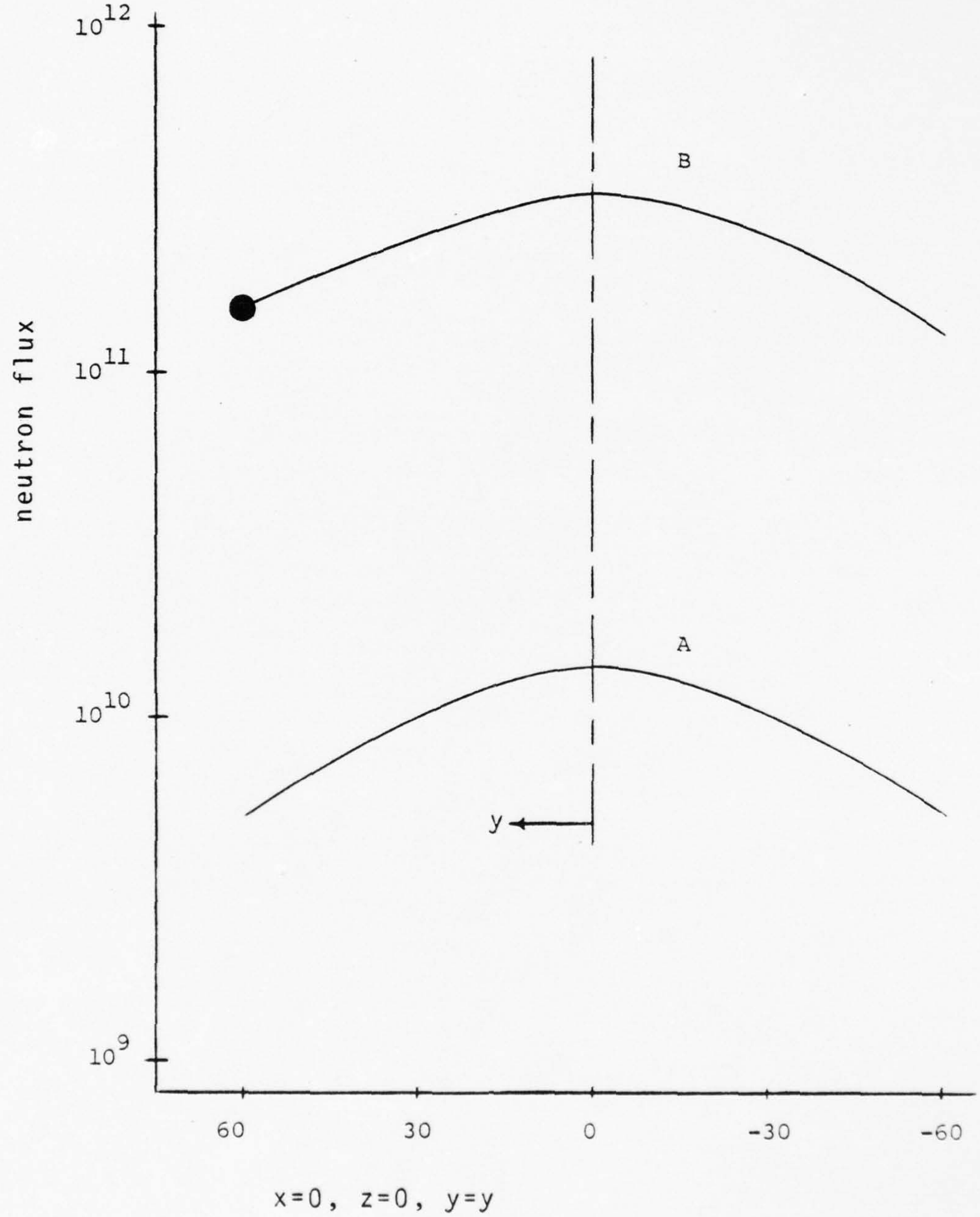
A - nonlinear neutron response
B - linear neutron response

Figure 22. Linear and nonlinear fluxes at (60,0,0) due to a 10 dollar/sec local perturbation at (0,60,40).



A - nonlinear neutron response
B - linear neutron response

Figure 23. Linear and nonlinear fluxes at (-60,0,80) due to a 10 dollar/sec local perturbation at (0,60,40).

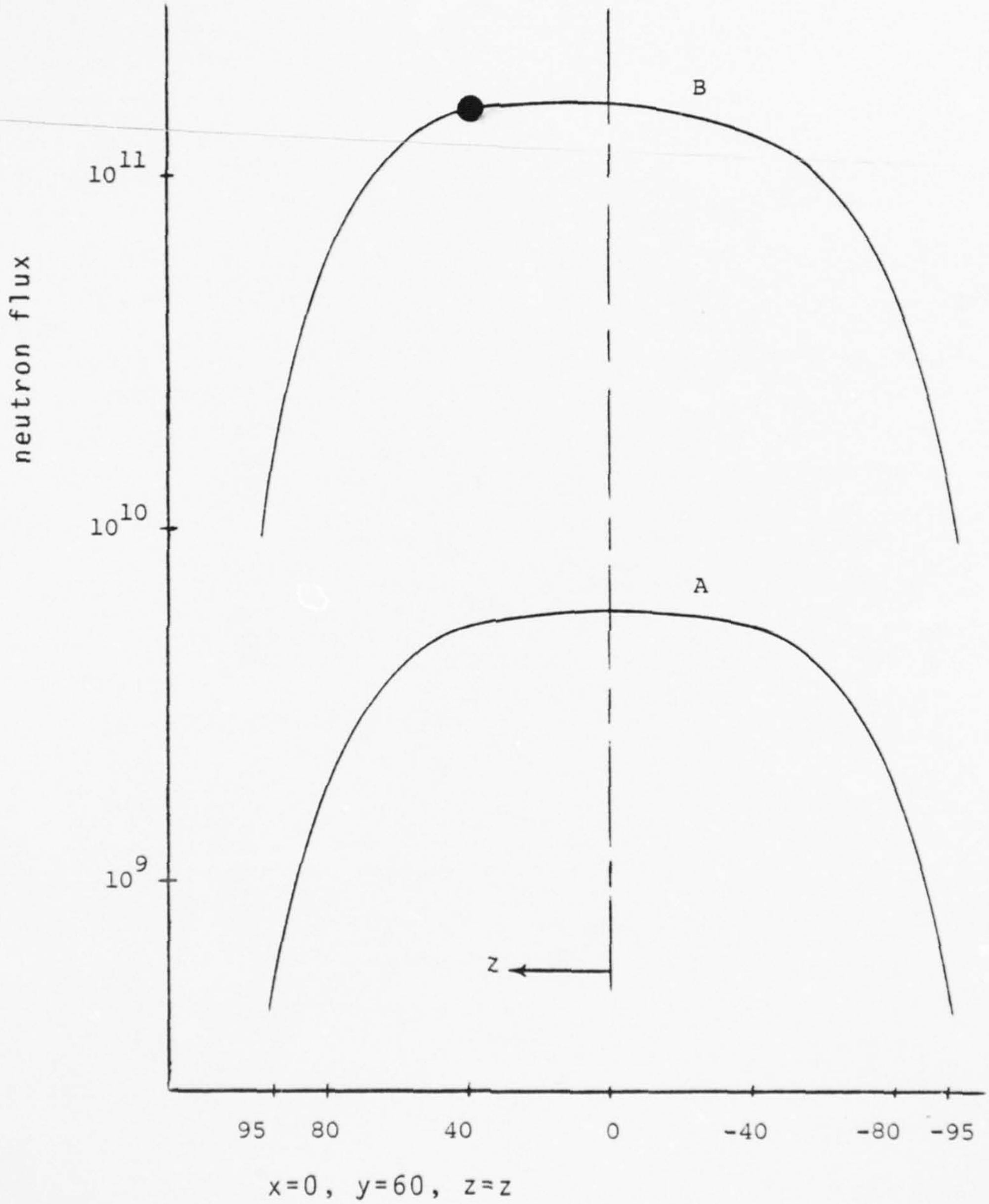


● - point of local perturbation

A - neutron flux at steady-state

B - neutron flux under local perturbation

Figure 24. Radial flux distribution for the steady state and 100 dollar/sec local perturbation.

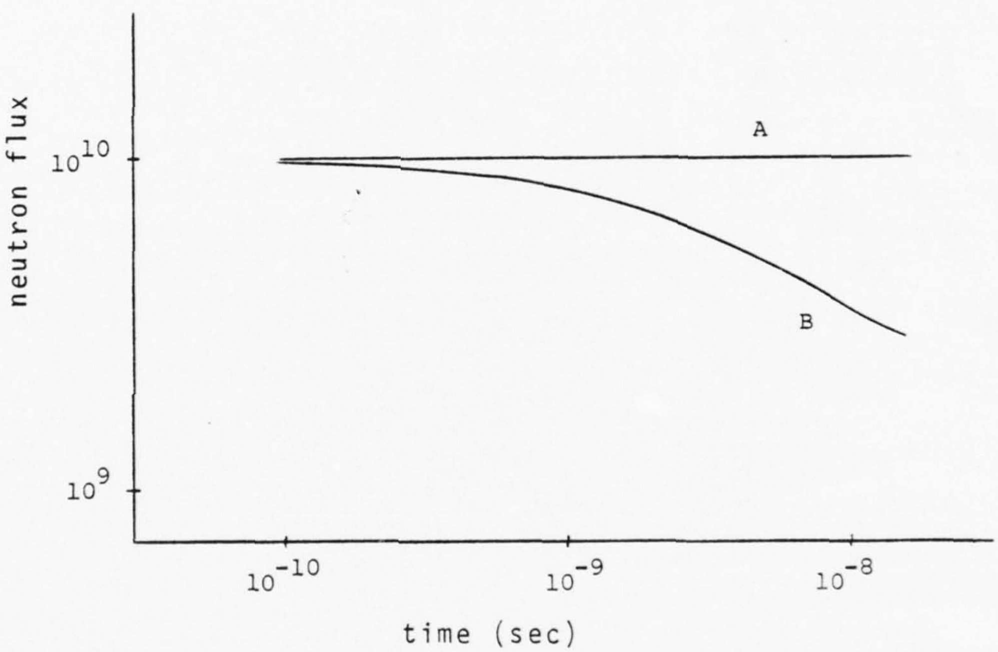


● - point of local perturbation

A - axial flux distribution during steady state

B - axial flux distribution during perturbation

Figure 25. Axial flux distribution for the steady state and 100 dollar/sec local perturbation.



A - mesh I
B - mesh II

Figure 26. Early time history of the neutron flux at core center using mesh I and mesh II.

and delayed neutrons on the flux. To obtain an idea of the difference in magnitude between the linear and nonlinear flux for the 100 dollar local perturbation, at time $t = 10^{-3}$ second, the linear case predicted a neutron flux at the core center of 2.024×10^{12} neutrons per cm^2 per sec. The nonlinear reactor predicted 1.85×10^{10} neutrons per cm^2 per sec. Although the numbers are not sufficiently accurate due to the crudeness of the finite element mesh, the significant difference in the order of magnitude between the linear and nonlinear neutron flux leads to the belief that the three-dimensional finite element model utilized is predicting the correct trend of neutron flux behavior.

The radial flux distribution for the steady state shown in figure 24 portrays the expected flux distribution. The radial flux distributions for the local perturbation case show a skew distribution at the point of perturbation. The axial flux distribution at steady state is very much symmetric about the center. Again, the expected skew at the point of perturbation is there, as shown in figure 25. Figure 26 gives an indication that the Σ_f^* for finer meshes is higher. However, curve B of figure 26 should be extended to longer times to verify this hypothesis. Curve B, as plotted in figure 26, used four hours of computer time employing the H-compiler of the IBM 360/67.

VI. CONCLUSIONS AND RECOMMENDATIONS

The finite element mesh employed here is crude, and, thus, results that were obtained should be considered as positive indicators rather than numerically conclusive facts. The trend of neutron flux behavior is the major thrust of this work. From the results obtained, it is concluded that the expected patterns of neutron behavior as predicted by the three-dimensional finite element model used do occur. These patterns were best demonstrated by the differences between the linear and nonlinear flux responses and by the spatial flux distribution at steady state and during local perturbation. The three-dimensional quadratic finite element model utilized should produce better results by resorting to a finer mesh. The draw-back to a finer mesh, of course, is the significant increase in computer time and storage requirements. Once accurate results are obtained through finer element meshes, comparisons between three- and two-dimensional models can be attempted.

A mesh generator was not developed for this problem. It is recommended that this be done to ease the transition from one mesh to another and to minimize human error. In addition, a similar calculation using a three-dimensional linear element should be performed to corroborate the results obtained here. It should be particularly noted that this type of problem is highly sensitive to the fission cross-section.

In the search for Σ_f^*, it was necessary to adjust Σ_f to the sixth decimal place. There is no exact method of deriving Σ_f^* due to the highly nonlinear aspect of the problem; therefore, trial and error must be used. Finally, the Gaussian quadrature used to determine the element matrices should be investigated. The cause of the differences in values obtained by using different number of integration points should be established. Was it due to the integration points, the coordinate transformation, or the element shape functions? This is an important question upon which future numerical results could be based.

APPENDIX A

MESH I CONNECTIVITY AND COORDINATES

The connectivity matrix for the 128-element mesh is as follows:

ELEMENT NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	3	12	11	10	2	26	28	27	35	37	46	45	44	36
2	1	4	14	13	12	3	26	29	28	35	38	48	47	46	37
3	1	5	16	15	14	4	26	30	29	35	39	50	49	48	38
4	1	6	18	17	16	5	26	31	30	35	40	52	51	50	39
5	1	7	20	19	18	6	26	32	31	35	41	54	53	52	40
6	1	8	22	21	20	7	26	33	32	35	42	56	55	54	41
7	1	9	24	23	22	8	26	34	33	35	43	58	57	56	42
8	1	2	10	25	24	9	26	27	34	35	36	44	59	58	43
9	35	37	46	45	44	36	60	62	61	69	71	80	79	78	70
10	35	38	48	47	46	37	60	63	62	69	72	82	81	80	71
11	35	39	50	49	48	38	60	64	63	69	73	84	83	82	72
12	35	40	52	51	50	39	60	65	64	69	74	86	85	84	73
13	35	41	54	53	52	40	60	66	65	69	75	88	87	86	74
14	35	42	56	55	54	41	60	67	66	69	76	90	89	88	75
15	35	43	58	57	56	42	60	68	67	69	77	92	91	90	76
16	35	36	44	59	58	43	60	61	68	69	70	78	93	92	77
17	69	71	80	79	78	70	118	120	119	425	427	436	435	434	426
18	69	72	82	81	80	71	118	121	120	425	428	438	437	436	427
19	69	73	84	83	82	72	118	122	121	425	429	440	439	438	428
20	69	74	86	85	84	73	118	123	122	425	430	442	441	440	429
21	69	75	88	87	86	74	118	124	123	425	431	444	443	442	430
22	69	76	90	89	88	75	118	125	124	425	432	446	445	444	431
23	69	77	92	91	90	76	118	126	125	425	433	448	447	446	432
24	69	70	78	93	92	77	118	119	126	425	426	434	449	448	433
25	78	95	379	378	377	94	119	410	409	434	451	476	475	474	450
26	379	95	78	79	80	96	410	119	120	476	451	434	435	436	452
27	80	97	381	380	379	96	120	411	410	436	453	478	477	476	452
28	80	98	383	382	381	97	120	412	411	436	454	480	479	478	453
29	383	98	80	81	82	99	412	120	121	480	454	436	437	438	455
30	82	100	385	384	383	99	121	413	412	438	456	482	481	480	455
31	82	101	387	386	385	100	121	414	413	438	457	484	483	482	456
32	387	101	82	83	84	102	414	121	122	484	457	438	439	440	458
33	84	103	389	388	387	102	122	415	414	440	459	486	485	484	458
34	84	104	391	390	389	103	122	416	415	440	460	488	487	486	459
35	391	104	84	85	86	105	416	122	123	488	460	440	441	442	461
36	86	106	393	392	391	105	123	417	416	442	462	490	489	488	461
37	86	107	395	394	393	106	123	418	417	442	463	492	491	490	462

38	395	107	86	87	88	108	418	123	124	492	463	442	443	444	464
39	88	109	397	396	395	108	124	419	418	444	465	494	493	492	464
40	88	110	399	398	397	109	124	420	419	444	466	496	495	494	465
41	399	110	88	89	90	111	420	124	125	496	466	444	445	446	467
42	90	112	401	400	399	111	125	421	420	446	468	498	497	496	467
43	90	113	403	402	401	112	125	422	421	446	469	500	499	498	468
44	403	113	90	91	92	114	422	125	126	500	469	446	447	448	470
45	92	115	405	404	403	114	126	423	422	448	471	502	501	500	470
46	92	116	407	406	405	115	126	424	423	448	472	504	503	502	471
47	407	116	92	93	78	117	424	126	119	504	472	448	449	434	473
48	78	94	377	408	407	117	119	409	424	434	450	474	505	504	473
49	44	128	331	330	329	127	61	362	361	78	95	379	378	377	94
50	331	128	44	45	46	129	362	61	62	379	95	78	79	80	96
51	46	130	333	332	331	129	62	363	362	80	97	381	380	379	96
52	46	131	335	334	333	130	62	364	363	80	98	383	382	381	97
53	335	131	45	47	48	132	364	62	63	383	98	80	81	82	99
54	48	133	337	336	335	132	63	365	364	82	100	385	384	383	99
55	48	134	339	338	337	133	63	366	365	82	101	387	386	385	100
56	339	134	48	49	50	135	366	63	64	387	101	82	83	84	102
57	50	136	341	340	339	135	64	367	366	84	103	389	388	387	102
58	50	137	343	342	341	136	64	368	367	84	104	391	390	389	103
59	343	137	50	51	52	138	368	64	65	391	104	84	85	86	105
60	52	139	345	344	343	138	65	369	368	86	106	393	392	391	105
61	52	140	347	346	345	139	65	370	369	86	107	395	394	393	106
62	347	140	52	53	54	141	370	65	66	395	107	86	87	88	108
63	54	142	349	348	347	141	66	371	370	88	109	397	396	395	108
64	54	143	351	350	349	142	66	372	371	88	110	399	398	397	109
65	351	143	54	55	56	144	372	66	67	399	110	88	89	90	111
66	56	145	353	352	351	144	67	373	372	90	112	401	400	399	111
67	56	145	355	354	353	145	67	374	373	90	113	403	402	401	112
68	355	146	56	57	58	147	374	67	68	403	113	90	91	92	114
69	58	148	357	356	355	147	68	375	374	92	115	405	404	403	114
70	58	149	359	358	357	148	68	376	375	92	116	407	406	405	115
71	359	149	58	59	44	150	376	68	61	407	116	92	93	78	117
72	44	127	329	360	359	150	61	361	376	78	94	377	408	407	117
73	10	152	283	282	281	151	27	314	313	44	128	331	330	329	127
74	283	152	10	11	12	153	314	27	28	331	128	44	45	46	129
75	12	154	285	284	283	153	28	315	314	46	130	333	332	331	129
76	12	155	287	286	285	154	28	316	315	46	131	335	334	333	130
77	287	155	12	13	14	156	316	28	29	335	131	46	47	48	132
78	14	157	289	288	287	156	29	317	316	48	133	337	336	335	132
79	14	158	291	290	289	157	29	318	317	48	134	339	338	337	133
80	291	158	14	15	16	159	318	29	30	339	134	48	49	50	135
81	16	160	293	292	291	159	30	319	318	50	136	341	340	339	135
82	16	161	295	294	293	160	30	320	319	50	137	343	342	341	136
83	295	161	16	17	18	162	320	30	31	343	137	50	51	52	138
84	18	163	297	296	295	162	31	321	320	52	139	345	344	343	138
85	18	164	299	298	297	163	31	322	321	52	140	347	346	345	139

86	299	164	18	19	20	165	322	31	32	347	140	52	53	54	141
87	20	166	301	300	299	165	32	323	322	54	142	349	348	347	141
88	20	167	303	302	301	166	32	324	323	54	143	351	350	349	142
89	303	167	20	21	22	168	324	32	33	351	143	54	55	56	144
90	22	169	305	304	303	168	33	325	324	56	145	353	352	351	144
91	22	170	307	306	305	169	33	326	325	56	146	355	354	353	145
92	307	170	22	23	24	171	326	33	34	355	146	56	57	58	147
93	24	172	309	308	307	171	34	327	326	58	148	357	356	355	147
94	24	173	311	310	309	172	34	328	327	58	149	359	358	357	148
95	311	173	24	25	10	174	328	34	27	359	149	58	59	44	150
96	10	151	281	312	311	174	27	313	328	44	127	329	360	359	150
97	184	186	195	194	193	185	175	177	176	1	3	12	11	10	2
98	184	187	197	196	195	186	175	178	177	1	4	14	13	12	3
99	184	188	199	198	197	187	175	179	178	1	5	16	15	14	4
100	184	189	201	200	199	188	175	180	179	1	6	18	17	16	5
101	184	190	203	202	201	189	175	181	180	1	7	20	19	18	6
102	184	191	205	204	203	190	175	182	181	1	8	22	21	20	7
103	184	192	207	206	205	191	175	183	182	1	9	24	23	22	8
104	184	185	193	208	207	192	175	176	183	1	2	10	25	24	9
105	193	210	235	234	233	209	176	266	265	10	152	283	282	281	151
106	235	210	193	194	195	211	266	175	177	283	152	10	11	12	153
107	195	212	237	236	235	211	177	267	266	12	154	285	284	283	153
108	195	213	239	238	237	212	177	268	267	12	155	287	286	285	154
109	239	213	195	196	197	214	268	177	178	287	155	12	13	14	156
110	197	215	241	240	239	214	178	269	268	14	157	289	288	287	156
111	197	216	243	242	241	215	178	270	269	14	158	291	290	289	157
112	243	216	197	198	199	217	270	178	179	291	158	14	15	16	159
113	199	218	245	244	243	217	179	271	270	16	160	293	292	291	159
114	199	219	247	246	245	218	179	272	271	16	161	295	294	293	160
115	247	219	199	200	201	220	272	179	180	295	161	16	17	18	162
116	201	221	249	248	247	220	180	273	272	18	163	297	296	295	162
117	201	222	251	250	249	221	180	274	273	18	164	299	298	297	163
118	251	222	201	202	203	223	274	180	181	299	164	18	19	20	165
119	203	224	253	252	251	223	181	275	274	20	166	301	300	299	165
120	203	225	255	254	253	224	181	276	275	20	167	303	302	301	166
121	255	225	203	204	205	226	276	181	182	303	167	20	21	22	168
122	205	227	257	256	255	226	182	277	276	22	169	305	304	303	168
123	205	228	259	258	257	227	182	278	277	22	170	307	306	305	169
124	259	228	205	206	207	229	278	182	183	307	170	22	23	24	171
125	207	230	261	260	259	229	183	279	278	24	172	309	308	307	171
126	207	231	263	262	261	230	183	280	279	24	173	311	310	309	172
127	263	231	207	208	193	232	280	183	176	311	173	24	25	10	174
128	193	209	233	264	263	232	176	265	280	10	151	281	312	311	174

The coordinates of the 128-element mesh are as follows:

NODE NUMBER	X	Y	Z
1	0.0	0.0	80.00
2	0.0	30.00	80.00
3	21.21	21.21	80.00
4	30.00	0.0	80.00
5	21.21	-21.21	80.00
6	0.0	-30.00	80.00
7	-21.21	-21.21	80.00
8	-30.00	0.0	80.00
9	-21.21	21.21	80.00
10	0.0	60.00	80.00
11	22.96	55.43	80.00
12	42.43	42.43	80.00
13	55.43	22.96	80.00
14	60.00	0.0	80.00
15	55.43	-22.96	80.00
16	42.43	-42.43	80.00
17	22.96	-55.43	80.00
18	0.0	-60.00	80.00
19	-22.96	-55.43	80.00
20	-42.43	-42.43	80.00
21	-55.43	-22.96	80.00
22	-60.00	0.0	80.00
23	-55.43	22.96	80.00
24	-42.43	42.43	80.00
25	-22.96	55.43	80.00
26	0.0	0.0	40.00
27	0.0	60.00	40.00
28	42.43	42.43	40.00
29	60.00	0.0	40.00
30	42.43	-42.43	40.00
31	0.0	-60.00	40.00
32	-42.43	-42.43	40.00
33	-60.00	0.0	40.00
34	-42.43	42.43	40.00
35	0.0	0.0	0.0
36	0.0	30.00	0.0
37	21.21	21.21	0.0
38	30.00	0.0	0.0
39	21.21	-21.21	0.0
40	0.0	-30.00	0.0
41	-21.21	-21.21	0.0
42	-30.00	0.0	0.0
43	-21.21	21.21	0.0
44	0.0	60.00	0.0

45	22.96	55.43	0.0
46	42.43	42.43	0.0
47	55.43	22.96	0.0
48	60.00	0.0	0.0
49	55.43	-22.96	0.0
50	42.43	-42.43	0.0
51	22.96	-55.43	0.0
52	0.0	-60.00	0.0
53	-22.96	-55.43	0.0
54	-42.43	-42.43	0.0
55	-55.43	-22.96	0.0
56	-60.00	0.0	0.0
57	-55.43	22.96	0.0
58	-42.43	42.43	0.0
59	-22.96	55.43	0.0
60	0.0	0.0	-40.00
61	0.0	60.00	-40.00
62	42.43	42.43	-40.00
63	60.00	0.0	-40.00
64	42.43	-42.43	-40.00
65	0.0	-60.00	-40.00
66	-42.43	-42.43	-40.00
67	-60.00	0.0	-40.00
68	-42.43	42.43	-40.00
69	0.0	0.0	-80.00
70	0.0	30.00	-80.00
71	21.21	21.21	-80.00
72	30.00	0.0	-80.00
73	21.21	-21.21	-80.00
74	0.0	-30.00	-80.00
75	-21.21	-21.21	-80.00
76	-30.00	0.0	-80.00
77	-21.21	21.21	-80.00
78	0.0	60.00	-80.00
79	22.96	55.43	-80.00
80	42.43	42.43	-80.00
81	55.43	22.96	-80.00
82	60.00	0.0	-80.00
83	55.43	-22.96	-80.00
84	42.43	-42.43	-80.00
85	22.96	-55.43	-80.00
86	0.0	-60.00	-80.00
87	-22.96	-55.43	-80.00
88	-42.43	-42.43	-80.00
89	-55.43	-22.96	-80.00
90	-60.00	0.0	-80.00
91	-55.43	22.96	-80.00
92	-42.43	42.43	-80.00

93	-22.96	55.43	-80.00
94	0.0	75.00	-80.00
95	16.80	71.60	-80.00
96	38.10	62.90	-80.00
97	53.03	53.03	-80.00
98	62.80	38.60	-80.00
99	71.30	17.00	-80.00
100	75.00	0.0	-80.00
101	71.30	-17.00	-80.00
102	62.80	-38.60	-80.00
103	53.03	-53.03	-80.00
104	38.10	-62.90	-80.00
105	17.10	-71.80	-80.00
106	0.0	-75.00	-80.00
107	-16.90	-71.80	-80.00
108	-38.50	-62.80	-80.00
109	-53.03	-53.03	-80.00
110	-62.70	-38.60	-80.00
111	-71.30	-17.10	-80.00
112	-75.00	0.0	-80.00
113	-71.30	17.10	-80.00
114	-62.90	38.30	-80.00
115	-53.03	53.03	-80.00
116	-38.60	62.80	-80.00
117	-17.30	71.30	-80.00
118	0.0	0.0	-95.00
119	0.0	60.00	-95.00
120	42.43	42.43	-95.00
121	60.00	0.0	-95.00
122	42.43	-42.43	-95.00
123	0.0	-60.00	-95.00
124	-42.43	-42.43	-95.00
125	-60.00	0.0	-95.00
126	-42.43	42.43	-95.00
127	0.0	75.00	0.0
128	16.80	71.60	0.0
129	38.10	62.90	0.0
130	53.03	53.03	0.0
131	62.80	38.60	0.0
132	71.30	17.00	0.0
133	75.00	0.0	0.0
134	71.30	-17.00	0.0
135	62.80	-38.60	0.0
136	53.03	-53.03	0.0
137	38.10	-62.90	0.0
138	17.10	-71.80	0.0
139	0.0	-75.00	0.0
140	-16.90	-71.80	0.0

141	-38.50	-62.80	0.0
142	-53.03	-53.03	0.0
143	-62.70	-38.60	0.0
144	-71.30	-17.10	0.0
145	-75.00	0.0	0.0
146	-71.30	17.10	0.0
147	-62.90	38.30	0.0
148	-53.03	53.03	0.0
149	-38.60	62.80	0.0
150	-17.30	71.30	0.0
151	0.0	75.00	80.00
152	16.80	71.60	80.00
153	38.10	62.90	80.00
154	53.03	53.03	80.00
155	62.80	38.60	80.00
156	71.30	17.00	80.00
157	75.00	0.0	80.00
158	71.30	-17.00	80.00
159	62.80	-38.60	80.00
160	53.03	-53.03	80.00
161	38.10	-62.90	80.00
162	17.10	-71.80	80.00
163	0.0	-75.00	80.00
164	-16.90	-71.80	80.00
165	-38.50	-62.80	80.00
166	-53.03	-53.03	80.00
167	-62.70	-38.60	80.00
168	-71.30	-17.10	80.00
169	-75.00	0.0	80.00
170	-71.30	17.10	80.00
171	-62.90	38.30	80.00
172	-53.03	53.03	80.00
173	-38.60	62.80	80.00
174	-17.30	71.30	80.00
175	0.0	0.0	95.00
176	0.0	60.00	95.00
177	42.43	42.43	95.00
178	60.00	0.0	95.00
179	42.43	-42.43	95.00
180	0.0	-60.00	95.00
181	-42.42	-42.43	95.00
182	-60.00	0.0	95.00
183	-42.43	42.43	95.00
184	0.0	0.0	110.00
185	0.0	30.00	110.00
186	21.21	21.21	110.00
187	30.00	0.0	110.00
188	21.21	-21.21	110.00

189	0.0	-30.00	110.00
190	-21.21	-21.21	110.00
191	-30.00	0.0	110.00
192	-21.21	21.21	110.00
193	0.0	60.00	110.00
194	22.96	55.43	110.00
195	42.43	42.43	110.00
196	55.43	22.96	110.00
197	60.00	0.0	110.00
198	55.43	-22.96	110.00
199	42.43	-42.43	110.00
200	22.96	-55.43	110.00
201	0.0	-60.00	110.00
202	-22.96	-55.43	110.00
203	-42.43	-42.43	110.00
204	-55.43	-22.96	110.00
205	-60.00	0.0	110.00
206	-55.43	22.96	110.00
207	-42.43	42.43	110.00
208	-22.96	55.43	110.00
209	0.0	75.00	110.00
210	16.80	71.60	110.00
211	38.10	62.90	110.00
212	53.03	53.03	110.00
213	62.80	38.60	110.00
214	71.30	17.00	110.00
215	75.00	0.0	110.00
216	71.30	-17.00	110.00
217	62.80	-38.60	110.00
218	53.03	-53.03	110.00
219	38.10	-62.90	110.00
220	17.10	-71.80	110.00
221	0.0	-75.00	110.00
222	-16.90	-71.80	110.00
223	-38.50	-62.80	110.00
224	-53.03	-53.03	110.00
225	-62.70	-38.60	110.00
226	-71.30	-17.10	110.00
227	-75.00	0.0	110.00
228	-71.30	17.10	110.00
229	-62.90	38.30	110.00
230	-53.03	53.03	110.00
231	-38.60	62.80	110.00
232	-17.30	71.30	110.00
233	0.0	90.00	110.00
234	17.56	88.27	110.00
235	34.44	83.15	110.00
236	50.00	74.83	110.00

237	63.64	63.64	110.00
238	74.83	50.00	110.00
239	83.15	34.44	110.00
240	88.27	17.56	110.00
241	90.00	0.0	110.00
242	88.27	-17.56	110.00
243	83.15	-34.44	110.00
244	74.83	-50.00	110.00
245	63.64	-63.64	110.00
246	50.00	-74.83	110.00
247	34.44	-83.15	110.00
248	17.56	-88.27	110.00
249	0.0	-90.00	110.00
250	-17.56	-88.27	110.00
251	-34.44	-83.15	110.00
252	-50.00	-74.83	110.00
253	-63.64	-63.64	110.00
254	-74.83	-50.00	110.00
255	-83.15	-34.44	110.00
256	-88.27	-17.56	110.00
257	-90.00	0.0	110.00
258	-88.27	17.56	110.00
259	-83.15	34.44	110.00
260	-74.83	50.00	110.00
261	-63.64	63.64	110.00
262	-50.00	74.83	110.00
263	-34.44	83.15	110.00
264	-17.56	88.27	110.00
265	0.0	90.00	95.00
266	34.44	83.15	95.00
267	63.64	63.64	95.00
268	83.15	34.44	95.00
269	90.00	0.0	95.00
270	83.15	-34.44	95.00
271	63.64	-63.64	95.00
272	34.44	-83.15	95.00
273	0.0	-90.00	95.00
274	-34.44	-83.15	95.00
275	-63.64	-63.64	95.00
276	-83.15	-34.44	95.00
277	-90.00	0.0	95.00
278	-83.15	34.44	95.00
279	-53.64	63.64	95.00
280	-34.44	83.15	95.00
281	0.0	90.00	80.00
282	17.56	88.27	80.00
283	34.44	83.15	80.00
284	50.00	74.83	80.00

AD-A038 775 NAVAL POSTGRADUATE SCHOOL MONTEREY CALIF F/G 18/11
FINITE ELEMENT SOLUTION OF A THREE-DIMENSIONAL NONLINEAR REACTO--ETC(U)
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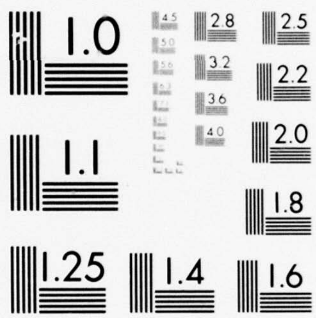

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

285	63.64	63.64	80.00
286	74.83	50.00	80.00
287	83.15	34.44	80.00
288	88.27	17.56	80.00
289	90.00	0.0	80.00
290	88.27	-17.56	80.00
291	83.15	-34.44	80.00
292	74.83	-50.00	80.00
293	63.64	-63.64	80.00
294	50.00	-74.83	80.00
295	34.44	-83.15	80.00
296	17.56	-88.27	80.00
297	0.0	-90.00	80.00
298	-17.56	-88.27	80.00
299	-34.44	-83.15	80.00
300	-50.00	-74.83	80.00
301	-63.64	-63.64	80.00
302	-74.83	-50.00	80.00
303	-83.15	-34.44	80.00
304	-88.27	-17.56	80.00
305	-90.00	0.0	80.00
306	-88.27	17.56	80.00
307	-83.15	34.44	80.00
308	-74.83	50.00	80.00
309	-63.64	63.64	80.00
310	-50.00	74.83	80.00
311	-34.44	83.15	80.00
312	-17.56	88.27	80.00
313	0.0	90.00	40.00
314	34.44	83.15	40.00
315	63.64	63.64	40.00
316	83.15	34.44	40.00
317	90.00	0.0	40.00
318	83.15	-34.44	40.00
319	63.64	-63.64	40.00
320	34.44	-83.15	40.00
321	0.0	-90.00	40.00
322	-34.44	-83.15	40.00
323	-63.64	-63.64	40.00
324	-83.15	-34.44	40.00
325	-90.00	0.0	40.00
326	-83.15	34.44	40.00
327	-63.64	63.64	40.00
328	-34.44	83.15	40.00
329	0.0	90.00	0.0
330	17.56	88.27	0.0
331	34.44	83.15	0.0
332	50.00	74.83	0.0

333	63.64	63.64	0.0
334	74.83	50.00	0.0
335	83.15	34.44	0.0
336	88.27	17.56	0.0
337	90.00	0.0	0.0
338	88.27	-17.56	0.0
339	83.15	-34.44	0.0
340	74.83	-50.00	0.0
341	63.64	-63.64	0.0
342	50.00	-74.83	0.0
343	34.44	-83.15	0.0
344	17.56	-88.27	0.0
345	0.0	-90.00	0.0
346	-17.56	-88.27	0.0
347	-34.44	-83.15	0.0
348	-50.00	-74.83	0.0
349	-63.64	-63.64	0.0
350	-74.83	-50.00	0.0
351	-83.15	-34.44	0.0
352	-88.27	-17.56	0.0
353	-90.00	0.0	0.0
354	-88.27	17.56	0.0
355	-83.15	34.44	0.0
356	-74.83	50.00	0.0
357	-63.64	63.64	0.0
358	-50.00	74.83	0.0
359	-34.44	83.15	0.0
360	-17.56	88.27	0.0
361	0.0	90.00	-40.00
362	34.44	83.15	-40.00
363	63.64	63.64	-40.00
364	83.15	34.44	-40.00
365	90.00	0.0	-40.00
366	83.15	-34.44	-40.00
367	63.64	-63.64	-40.00
368	34.44	-83.15	-40.00
369	0.0	-90.00	-40.00
370	-34.44	-83.15	-40.00
371	-63.64	-63.64	-40.00
372	-83.15	-34.44	-40.00
373	-90.00	0.0	-40.00
374	-83.15	34.44	-40.00
375	-63.64	63.64	-40.00
376	-34.44	83.15	-40.00
377	0.0	90.00	-80.00
378	17.56	88.27	-80.00
379	34.44	83.15	-80.00
380	50.00	74.83	-80.00

381	63.64	63.64	-80.00
382	74.83	50.00	-80.00
383	83.15	34.44	-80.00
384	88.27	17.56	-80.00
385	90.00	0.0	-80.00
386	88.27	-17.56	-80.00
387	83.15	-34.44	-80.00
388	74.83	-50.00	-80.00
389	63.64	-63.64	-80.00
390	50.00	-74.83	-80.00
391	34.44	-83.15	-80.00
392	17.56	-88.27	-80.00
393	0.0	-90.00	-80.00
394	-17.56	-88.27	-80.00
395	-34.44	-83.15	-80.00
396	-50.00	-74.83	-80.00
397	-63.64	-63.64	-80.00
398	-74.83	-50.00	-80.00
399	-83.15	-34.44	-80.00
400	-88.27	-17.56	-80.00
401	-90.00	0.0	-80.00
402	-88.27	17.56	-80.00
403	-83.15	34.44	-80.00
404	-74.83	50.00	-80.00
405	-63.64	63.64	-80.00
406	-50.00	74.83	-80.00
407	-34.44	83.15	-80.00
408	-17.56	88.27	-80.00
409	0.0	90.00	-95.00
410	34.44	83.15	-95.00
411	63.64	63.64	-95.00
412	83.15	34.44	-95.00
413	90.00	0.0	-95.00
414	83.15	-34.44	-95.00
415	63.64	-63.64	-95.00
416	34.44	-83.15	-95.00
417	0.0	-90.00	-95.00
418	-34.44	-83.15	-95.00
419	-63.64	-63.64	-95.00
420	-83.15	-34.44	-95.00
421	-90.00	0.0	-95.00
422	-83.15	34.44	-95.00
423	-63.64	63.64	-95.00
424	-34.44	83.15	-95.00
425	0.0	0.0	-110.00
426	0.0	30.00	-110.00
427	21.21	21.21	-110.00
428	30.00	0.0	-110.00

429	21.21	-21.21	-110.00
430	0.0	-30.00	-110.00
431	-21.21	-21.21	-110.00
432	-30.00	0.0	-110.00
433	-21.21	21.21	-110.00
434	0.0	60.00	-110.00
435	22.96	55.43	-110.00
436	42.43	42.43	-110.00
437	55.43	22.96	-110.00
438	60.00	0.0	-110.00
439	55.43	-22.96	-110.00
440	42.43	-42.43	-110.00
441	22.96	-55.43	-110.00
442	0.0	-60.00	-110.00
443	-22.96	-55.43	-110.00
444	-42.43	-42.43	-110.00
445	-55.43	-22.96	-110.00
446	-60.00	0.0	-110.00
447	-55.43	22.96	-110.00
448	-42.43	42.43	-110.00
449	-22.96	55.43	-110.00
450	0.0	75.00	-110.00
451	16.80	71.60	-110.00
452	38.10	62.90	-110.00
453	53.03	53.03	-110.00
454	62.80	38.60	-110.00
455	71.30	17.00	-110.00
456	75.00	0.0	-110.00
457	71.30	-17.00	-110.00
458	62.80	-38.60	-110.00
459	53.03	-53.03	-110.00
460	38.10	-62.90	-110.00
461	17.10	-71.80	-110.00
462	0.0	-75.00	-110.00
463	-16.90	-71.80	-110.00
464	-38.50	-62.80	-110.00
465	-53.03	-53.03	-110.00
466	-62.70	-38.60	-110.00
467	-71.30	-17.10	-110.00
468	-75.00	0.0	-110.00
469	-71.30	17.10	-110.00
470	-62.90	38.30	-110.00
471	-53.03	53.03	-110.00
472	-38.60	62.80	-110.00
473	-17.30	71.30	-110.00
474	0.0	90.00	-110.00
475	17.56	88.27	-110.00
476	34.44	83.15	-110.00

477	50.00	74.83	-110.00
478	63.64	63.64	-110.00
479	74.83	50.00	-110.00
480	83.15	34.44	-110.00
481	88.27	17.56	-110.00
482	90.00	0.0	-110.00
483	88.27	-17.56	-110.00
484	83.15	-34.44	-110.00
485	74.83	-50.00	-110.00
486	63.64	-63.64	-110.00
487	50.00	-74.83	-110.00
488	34.44	-83.15	-110.00
489	17.56	-88.27	-110.00
490	0.0	-90.00	-110.00
491	-17.56	-88.27	-110.00
492	-34.44	-83.15	-110.00
493	-50.00	-74.83	-110.00
494	-63.64	-63.64	-110.00
495	-74.83	-50.00	-110.00
496	-83.15	-34.44	-110.00
497	-88.27	-17.56	-110.00
498	-90.00	0.0	-110.00
499	-88.27	17.56	-110.00
500	-83.15	34.44	-110.00
501	-74.83	50.00	-110.00
502	-63.64	63.64	-110.00
503	-50.00	74.83	-110.00
504	-34.44	83.15	-110.00
505	-17.56	88.27	-110.00

APPENDIX B

MESH II CONNECTIVITY AND COORDINATES

The connectivity matrix for the 192-element mesh is as follows:

ELEMENT NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	3	12	11	10	2	26	28	27	35	37	46	45	44	36
2	1	4	14	13	12	3	26	29	28	35	38	48	47	46	37
3	1	5	16	15	14	4	26	30	29	35	39	50	49	48	38
4	1	6	18	17	16	5	26	31	30	35	40	52	51	50	39
5	1	7	20	19	18	6	26	32	31	35	41	54	53	52	40
6	1	8	22	21	20	7	26	33	32	35	42	56	55	54	41
7	1	9	24	23	22	8	26	34	33	35	43	58	57	56	42
8	1	2	10	25	24	9	26	27	34	35	36	44	59	58	43
9	35	37	46	45	44	36	60	62	61	69	71	80	79	78	70
10	35	38	48	47	46	37	60	63	62	69	72	82	81	80	71
11	35	39	50	49	48	38	60	64	63	69	73	84	83	82	72
12	35	40	52	51	50	39	60	65	64	69	74	86	85	84	73
13	35	41	54	53	52	40	60	66	65	69	75	88	87	86	74
14	35	42	56	55	54	41	60	67	66	69	76	90	89	88	75
15	35	43	58	57	56	42	60	68	67	69	77	92	91	90	76
16	35	36	44	59	58	43	60	61	68	69	70	78	93	92	77
17	69	71	80	79	78	70	94	96	95	103	105	114	113	112	104
18	69	72	82	81	80	71	94	97	96	103	106	116	115	114	105
19	69	73	84	83	82	72	94	98	97	103	107	118	117	116	106
20	69	74	86	85	84	73	94	99	98	103	108	120	119	118	107
21	69	75	88	87	86	74	94	100	99	103	109	122	121	120	108
22	69	76	90	89	88	75	94	101	100	103	110	124	123	122	109
23	69	77	92	91	90	76	94	102	101	103	111	126	125	124	110
24	69	70	78	93	92	77	94	95	102	103	104	112	127	126	111
25	103	105	114	113	112	104	128	130	129	137	139	148	147	146	138
26	103	106	116	115	114	105	128	131	130	137	140	150	149	148	139
27	103	107	118	117	116	106	128	132	131	137	141	152	151	150	140
28	103	108	120	119	118	107	128	133	132	137	142	154	153	152	141
29	103	109	122	121	120	108	128	134	133	137	143	156	155	154	142
30	103	110	124	123	122	109	128	135	134	137	144	158	157	156	143
31	103	111	126	125	124	110	128	136	135	137	145	160	159	158	144
32	103	104	112	127	126	111	128	129	136	137	138	146	161	160	145
33	137	139	148	147	146	138	162	164	163	300	302	311	310	309	301
34	137	140	150	149	148	139	162	165	164	300	303	313	312	311	302
35	137	141	152	151	150	140	162	166	165	300	304	315	314	313	303

36	137	142	154	153	152	141	162	167	166	300	305	317	316	315	304
37	137	143	156	155	154	142	162	168	167	300	306	319	318	317	305
38	137	144	158	157	156	143	162	169	168	300	307	321	320	319	306
39	137	145	160	159	158	144	162	170	169	300	308	323	322	321	307
40	137	138	146	161	160	145	162	163	170	300	301	309	324	323	308
41	717	710	695	694	693	709	171	173	172	1	3	12	11	10	2
42	717	711	697	696	695	710	171	174	173	1	4	14	13	12	3
43	717	712	699	698	697	711	171	175	174	1	5	16	15	14	4
44	717	713	701	700	699	712	171	176	175	1	6	18	17	16	5
45	717	714	703	702	701	713	171	177	176	1	7	20	19	18	6
46	717	715	705	704	703	714	171	178	177	1	8	22	21	20	7
47	717	716	707	706	705	715	171	179	178	1	9	24	23	22	8
48	717	709	693	708	707	716	171	172	179	1	2	10	25	24	9
49	693	670	639	638	637	669	172	622	621	10	181	591	590	589	180
50	639	670	693	694	695	671	622	172	173	591	181	10	11	12	182
51	695	672	641	640	639	671	173	623	622	12	183	593	592	591	182
52	695	673	643	642	641	672	173	624	623	12	184	595	594	593	183
53	643	673	695	696	697	674	624	173	174	595	184	12	13	14	185
54	697	675	645	644	643	674	174	625	624	14	186	597	596	595	185
55	697	676	647	646	645	675	174	626	625	14	187	599	598	597	186
56	647	676	697	698	699	677	626	174	175	599	187	14	15	16	188
57	699	678	649	648	647	677	175	627	626	16	189	601	600	599	188
58	699	679	651	650	649	678	175	628	627	16	190	603	602	601	189
59	651	679	699	700	701	680	628	175	176	603	190	16	17	18	191
60	701	681	653	652	651	680	176	629	628	18	192	605	604	603	191
61	701	682	655	654	653	681	176	630	629	18	193	607	606	605	192
62	655	682	701	702	703	683	630	176	177	607	193	18	19	20	194
63	703	684	657	656	655	683	177	631	630	20	195	609	608	607	194
64	703	685	659	658	657	684	177	632	631	20	196	611	610	609	195
65	659	685	703	704	705	686	632	177	178	611	196	20	21	22	197
66	705	687	661	660	659	686	178	633	632	22	198	613	612	611	197
67	705	688	663	662	661	687	178	634	633	22	199	615	614	613	198
68	663	688	705	706	707	689	634	178	179	615	199	22	23	24	200
69	707	690	665	664	663	689	179	635	634	24	201	617	616	615	200
70	707	691	667	666	665	690	179	636	635	24	202	619	618	617	201
71	667	691	707	708	693	692	636	179	172	619	202	24	25	10	203
72	693	669	637	668	667	692	172	621	636	10	180	589	620	619	203
73	10	181	591	590	589	180	27	574	573	44	205	543	542	541	204
74	591	181	10	11	12	182	574	27	28	543	205	44	45	46	206
75	12	183	593	592	591	182	28	575	574	46	207	545	544	543	206
76	12	184	595	594	593	183	28	576	575	46	208	547	546	545	207
77	595	184	12	13	14	185	576	28	29	547	208	46	47	48	209
78	14	186	597	596	595	185	29	577	576	48	210	549	548	547	209
79	14	187	599	598	597	186	29	578	577	48	211	551	550	549	210
80	599	187	14	15	16	188	578	29	30	551	211	48	49	50	212
81	16	189	601	600	599	188	30	579	578	50	213	553	552	551	212
82	16	190	603	602	601	189	30	580	579	50	214	555	554	553	213
83	603	190	16	17	18	191	580	30	31	555	214	50	51	52	215

84	18	192	605	604	603	191	31	581	580	52	216	557	556	555	215
85	18	193	607	606	605	192	31	582	581	52	217	559	558	557	216
86	607	193	13	19	20	194	582	31	32	559	217	52	53	54	218
87	20	195	609	608	607	194	32	583	582	54	219	561	560	559	218
88	20	196	611	610	609	195	32	584	583	54	220	563	562	561	219
89	611	196	20	21	22	197	584	32	33	563	220	54	55	56	221
90	22	198	613	612	611	197	33	585	584	56	222	565	564	563	221
91	22	199	615	614	613	198	33	586	585	56	223	567	566	565	222
92	615	199	22	23	24	200	586	33	34	567	223	56	57	58	224
93	24	201	617	616	615	200	34	587	586	58	225	569	568	567	224
94	24	202	619	618	617	201	34	588	587	58	226	571	570	569	225
95	619	202	24	25	10	203	588	34	27	571	226	58	59	44	227
96	10	180	589	620	619	203	27	573	588	44	204	541	572	571	227
97	44	205	543	542	541	204	61	525	525	78	229	495	494	493	228
98	543	205	44	45	46	206	526	61	62	495	229	78	79	80	230
99	46	207	545	544	543	206	62	527	526	80	231	497	496	495	230
100	46	208	547	546	545	207	62	528	527	80	232	499	498	497	231
101	547	208	46	47	48	209	528	62	63	499	232	80	81	82	233
102	48	210	549	548	547	209	63	529	528	82	234	501	500	499	233
103	48	211	551	550	549	210	63	530	529	82	235	503	502	501	234
104	551	211	48	49	50	212	530	63	64	503	235	82	83	84	236
105	50	213	553	552	551	212	64	531	530	84	237	505	504	503	236
106	50	214	555	554	553	213	64	532	531	84	238	507	506	505	237
107	555	214	50	51	52	215	532	64	65	507	238	84	85	86	239
108	52	216	557	556	555	215	65	533	532	86	240	509	508	507	239
109	52	217	559	558	557	216	65	534	533	86	241	511	510	509	240
110	559	217	52	53	54	218	534	65	66	511	241	86	87	88	242
111	54	219	561	560	559	218	66	535	534	88	243	513	512	511	242
112	54	220	563	562	561	219	66	536	535	88	244	515	514	513	243
113	563	220	54	55	56	221	536	65	67	515	244	88	89	90	245
114	56	222	565	564	563	221	67	537	536	90	246	517	516	515	245
115	56	223	567	566	565	222	67	538	537	90	247	519	518	517	246
116	567	223	56	57	58	224	538	67	68	519	247	90	91	92	248
117	58	225	569	568	567	224	68	539	538	92	249	521	520	519	248
118	58	226	571	570	569	225	68	540	539	92	250	523	522	521	249
119	571	226	58	59	44	227	540	68	61	523	250	92	93	78	251
120	44	204	541	572	571	227	61	525	540	78	228	493	524	523	251
121	78	229	495	494	493	228	95	478	477	112	253	447	446	445	252
122	495	229	78	79	80	230	478	95	96	447	253	112	113	114	254
123	80	231	497	496	495	230	96	479	478	114	255	449	448	447	254
124	80	232	499	498	497	231	96	480	479	114	256	451	450	449	255
125	499	232	80	81	82	233	480	96	97	451	256	114	115	116	257
126	82	234	501	500	499	233	97	481	480	116	258	453	452	451	257
127	82	235	503	502	501	234	97	482	481	116	259	455	454	453	258
128	503	235	82	83	84	236	482	97	98	455	259	116	117	118	260
129	84	237	505	504	503	236	98	483	482	118	261	457	456	455	260
130	84	238	507	506	505	237	98	484	483	118	262	459	458	457	261
131	507	238	84	85	86	239	484	98	99	459	262	118	119	120	263

132	86	240	509	508	507	239	99	485	484	120	264	461	460	459	263
133	86	241	511	510	509	240	99	486	485	120	265	463	462	461	264
134	511	241	86	87	88	242	486	99	100	463	265	120	121	122	266
135	88	243	513	512	511	242	100	487	486	122	267	465	464	463	266
136	88	244	515	514	513	243	100	488	487	122	268	467	466	465	267
137	515	244	88	89	90	245	488	100	101	467	268	122	123	124	269
138	90	246	517	516	515	245	101	489	488	124	270	469	468	467	269
139	90	247	519	518	517	246	101	490	489	124	271	471	470	469	270
140	519	247	90	91	92	248	490	101	102	471	271	124	125	126	272
141	92	249	521	520	519	248	102	491	490	126	273	473	472	471	272
142	92	250	523	522	521	249	102	492	491	126	274	475	474	473	273
143	523	250	92	93	78	251	492	102	95	475	274	126	127	112	275
144	78	228	493	524	523	251	95	477	492	112	252	445	476	475	275
145	112	253	447	446	445	252	129	430	429	146	277	399	398	397	276
146	447	253	112	113	114	254	430	129	130	399	277	146	147	148	278
147	114	255	449	448	447	254	130	431	430	148	279	401	400	399	278
148	114	256	451	450	449	255	130	432	431	148	280	403	402	401	279
149	451	256	114	115	116	257	432	130	131	403	280	148	149	150	281
150	116	258	453	452	451	257	131	433	432	150	282	405	404	403	281
151	116	259	455	454	453	258	131	434	433	150	283	407	406	405	282
152	455	259	116	117	118	260	434	131	132	407	283	150	151	152	284
153	118	261	457	456	455	260	132	435	434	152	285	409	408	407	284
154	118	262	459	458	457	261	132	436	435	152	286	411	410	409	285
155	459	262	118	119	120	263	436	132	133	411	286	152	153	154	287
156	120	264	461	460	459	263	133	437	436	154	288	413	412	411	287
157	120	265	463	462	461	264	133	438	437	154	289	415	414	413	288
158	463	265	120	121	122	266	438	133	134	415	289	154	155	156	290
159	122	267	465	464	463	266	134	439	438	156	291	417	416	415	290
160	122	268	467	466	465	267	134	440	439	156	292	419	418	417	291
161	467	268	122	123	124	269	440	134	135	419	292	156	157	158	293
162	124	270	469	468	467	269	135	441	440	158	294	421	420	419	293
163	124	271	471	470	469	270	135	442	441	158	295	423	422	421	294
164	471	271	124	125	126	272	442	135	136	423	295	158	159	160	296
165	126	273	473	472	471	272	136	443	442	160	297	425	424	423	296
166	126	274	475	474	473	273	136	444	443	160	298	427	426	425	297
167	475	274	126	127	112	275	444	136	129	427	298	160	161	146	299
168	112	252	445	476	475	275	129	429	444	146	276	397	428	427	299
169	146	277	399	398	397	276	163	382	381	309	326	351	350	349	325
170	399	277	146	147	148	278	382	163	164	351	326	309	310	311	327
171	148	279	401	400	399	278	164	383	382	311	328	353	352	351	327
172	148	280	403	402	401	279	164	384	383	311	329	355	354	353	328
173	403	280	148	149	150	281	384	164	165	355	329	311	312	313	330
174	150	282	405	404	403	281	165	385	384	313	331	357	356	355	330
175	150	283	407	406	405	282	165	386	385	313	332	359	358	357	331
176	407	283	150	151	152	284	386	165	166	359	332	313	314	315	333
177	152	285	409	408	407	284	166	387	386	315	334	361	360	359	333
178	152	286	411	410	409	285	166	388	387	315	335	363	362	361	334
179	411	286	152	153	154	287	388	166	167	363	335	315	316	317	336

180	154	288	413	412	411	287	167	389	388	317	337	365	364	363	336
181	154	289	415	414	413	288	167	390	389	317	338	367	366	365	337
182	415	289	154	155	156	290	390	167	168	367	338	317	318	319	339
183	156	291	417	416	415	290	168	391	390	319	340	369	368	367	339
184	156	292	419	418	417	291	168	392	391	319	341	371	370	369	340
185	419	292	156	157	158	293	392	168	169	371	341	319	[illegible]	321	342
186	158	294	421	420	419	293	169	393	392	321	343	373	[illegible]	371	342
187	158	295	423	422	421	294	169	394	393	321	344	375	374	373	343
188	423	295	158	159	160	296	394	169	170	375	344	321	322	323	345
189	160	297	425	424	423	296	170	395	394	323	346	377	376	375	345
190	160	298	427	426	425	297	170	396	395	323	347	379	378	377	346
191	427	298	160	161	146	299	396	170	163	379	347	323	324	309	348
192	146	276	397	428	427	299	163	381	396	309	325	349	380	379	348

The coordinates of the 192-element mesh are as follows:

NODE NUMBER	X	Y	Z
1	0.0	0.0	80.00
2	0.0	30.00	80.00
3	21.21	21.21	80.00
4	30.00	0.0	80.00
5	21.21	-21.21	80.00
6	0.0	-30.00	80.00
7	-21.21	-21.21	80.00
8	-30.00	0.0	80.00
9	-21.21	21.21	80.00
10	0.0	60.00	80.00
11	22.96	55.43	80.00
12	42.43	42.43	80.00
13	55.43	22.96	80.00
14	60.00	0.0	80.00
15	55.43	-22.96	80.00
16	42.43	-42.43	80.00
17	22.96	-55.43	80.00
18	0.0	-60.00	80.00
19	-22.96	-55.43	80.00
20	-42.43	-42.43	80.00
21	-55.43	-22.96	80.00
22	-60.00	0.0	80.00
23	-55.43	22.96	80.00
24	-42.43	42.43	80.00
25	-22.96	55.43	80.00
26	0.0	0.0	60.00

27	0.0	60.00	60.00
28	42.43	42.43	60.00
29	60.00	0.0	60.00
30	42.43	-42.43	60.00
31	0.0	-60.00	60.00
32	-42.42	-42.43	60.00
33	-60.00	0.0	60.00
34	-42.43	42.43	60.00
35	0.0	0.0	40.00
36	0.0	30.00	40.00
37	21.21	21.21	40.00
38	30.00	0.0	40.00
39	21.21	-21.21	40.00
40	0.0	-30.00	40.00
41	-21.21	-21.21	40.00
42	-30.00	0.0	40.00
43	-21.21	21.21	40.00
44	0.0	60.00	40.00
45	22.96	55.43	40.00
46	42.43	42.43	40.00
47	55.43	22.96	40.00
48	60.00	0.0	40.00
49	55.43	-22.96	40.00
50	42.43	-42.43	40.00
51	22.96	-55.43	40.00
52	0.0	-60.00	40.00
53	-22.96	-55.43	40.00
54	-42.43	-42.43	40.00
55	-55.43	-22.96	40.00
56	-60.00	0.0	40.00
57	-55.43	22.96	40.00
58	-42.43	42.43	40.00
59	-22.96	55.43	40.00
60	0.0	0.0	20.00
61	0.0	60.00	20.00
62	42.43	42.43	20.00
63	60.00	0.0	20.00
64	42.43	-42.43	20.00
65	0.0	-60.00	20.00
66	-42.42	-42.43	20.00
67	-60.00	0.0	20.00
68	-42.43	42.43	20.00
69	0.0	0.0	0.0
70	0.0	30.00	0.0
71	21.21	21.21	0.0
72	30.00	0.0	0.0
73	21.21	-21.21	0.0
74	0.0	-30.00	0.0

75	-21.21	-21.21	0.0
76	-30.00	0.0	0.0
77	-21.21	21.21	0.0
78	0.0	60.00	0.0
79	22.96	55.43	0.0
80	42.43	42.43	0.0
81	55.43	22.96	0.0
82	60.00	0.0	0.0
83	55.43	-22.96	0.0
84	42.43	-42.43	0.0
85	22.96	-55.43	0.0
86	0.0	-60.00	0.0
87	-22.96	-55.43	0.0
88	-42.43	-42.43	0.0
89	-55.43	-22.96	0.0
90	-60.00	0.0	0.0
91	-55.43	22.96	0.0
92	-42.43	42.43	0.0
93	-22.96	55.43	0.0
94	0.0	0.0	-20.00
95	0.0	60.00	-20.00
96	42.43	42.43	-20.00
97	60.00	0.0	-20.00
98	42.43	-42.43	-20.00
99	0.0	-60.00	-20.00
100	-42.42	-42.43	-20.00
101	-60.00	0.0	-20.00
102	-42.43	42.43	-20.00
103	0.0	0.0	-40.00
104	0.0	30.00	-40.00
105	21.21	21.21	-40.00
106	30.00	0.0	-40.00
107	21.21	-21.21	-40.00
108	0.0	-30.00	-40.00
109	-21.21	-21.21	-40.00
110	-30.00	0.0	-40.00
111	-21.21	21.21	-40.00
112	0.0	60.00	-40.00
113	22.96	55.43	-40.00
114	42.43	42.43	-40.00
115	55.43	22.96	-40.00
116	60.00	0.0	-40.00
117	55.43	-22.96	-40.00
118	42.43	-42.43	-40.00
119	22.96	-55.43	-40.00
120	0.0	-60.00	-40.00
121	-22.96	-55.43	-40.00
122	-42.43	-42.43	-40.00

123	-55.43	-22.96	-40.00
124	-60.00	0.0	-40.00
125	-55.43	22.96	-40.00
126	-42.43	42.43	-40.00
127	-22.96	55.43	-40.00
128	0.0	0.0	-60.00
129	0.0	60.00	-60.00
130	42.43	42.43	-60.00
131	60.00	0.0	-60.00
132	42.43	-42.43	-60.00
133	0.0	-60.00	-60.00
134	-42.42	-42.43	-60.00
135	-60.00	0.0	-60.00
136	-42.43	42.43	-60.00
137	0.0	0.0	-80.00
138	0.0	30.00	-80.00
139	21.21	21.21	-80.00
140	30.00	0.0	-80.00
141	21.21	-21.21	-80.00
142	0.0	-30.00	-80.00
143	-21.21	-21.21	-80.00
144	-30.00	0.0	-80.00
145	-21.21	21.21	-80.00
146	0.0	60.00	-80.00
147	22.96	55.43	-80.00
148	42.43	42.43	-80.00
149	55.43	22.96	-80.00
150	60.00	0.0	-80.00
151	55.43	-22.96	-80.00
152	42.43	-42.43	-80.00
153	22.96	-55.43	-80.00
154	0.0	-60.00	-80.00
155	-22.96	-55.43	-80.00
156	-42.43	-42.43	-80.00
157	-55.43	-22.96	-80.00
158	-60.00	0.0	-80.00
159	-55.43	22.96	-80.00
160	-42.43	42.43	-80.00
161	-22.96	55.43	-80.00
162	0.0	0.0	-95.00
163	0.0	60.00	-95.00
164	42.43	42.43	-95.00
165	60.00	0.0	-95.00
166	42.43	-42.43	-95.00
167	0.0	-60.00	-95.00
168	-42.42	-42.43	-95.00
169	-60.00	0.0	-95.00
170	-42.43	42.43	-95.00

171	0.0	0.0	95.00
172	0.0	60.00	95.00
173	42.43	42.43	95.00
174	60.00	0.0	95.00
175	42.43	-42.43	95.00
176	0.0	-60.00	95.00
177	-42.42	-42.43	95.00
178	-60.00	0.0	95.00
179	-42.43	42.43	95.00
180	0.0	75.00	80.00
181	16.80	71.60	80.00
182	38.10	62.90	80.00
183	53.03	53.03	80.00
184	62.80	38.60	80.00
185	71.30	17.00	80.00
186	75.00	0.0	80.00
187	71.30	-17.00	80.00
188	62.80	-38.60	80.00
189	53.03	-53.03	80.00
190	38.10	-62.90	80.00
191	17.10	-71.80	80.00
192	0.0	-75.00	80.00
193	-16.90	-71.80	80.00
194	-38.50	-62.80	80.00
195	-53.03	-53.03	80.00
196	-62.70	-38.60	80.00
197	-71.30	-17.10	80.00
198	-75.00	0.0	80.00
199	-71.30	17.10	80.00
200	-62.90	38.30	80.00
201	-53.03	53.03	80.00
202	-38.60	62.80	80.00
203	-17.30	71.30	80.00
204	0.0	75.00	40.00
205	16.80	71.60	40.00
206	38.10	62.90	40.00
207	53.03	53.03	40.00
208	62.80	38.60	40.00
209	71.30	17.00	40.00
210	75.00	0.0	40.00
211	71.30	-17.00	40.00
212	62.80	-38.60	40.00
213	53.03	-53.03	40.00
214	38.10	-62.90	40.00
215	17.10	-71.80	40.00
216	0.0	-75.00	40.00
217	-16.90	-71.80	40.00
218	-38.50	-62.80	40.00

219	-53.03	-53.03	40.00
220	-62.70	-38.60	40.00
221	-71.30	-17.10	40.00
222	-75.00	0.0	40.00
223	-71.30	17.10	40.00
224	-62.90	38.30	40.00
225	-53.03	53.03	40.00
226	-38.60	62.80	40.00
227	-17.30	71.30	40.00
228	0.0	75.00	0.0
229	16.80	71.60	0.0
230	38.10	62.90	0.0
231	53.03	53.03	0.0
232	62.80	38.60	0.0
233	71.30	17.00	0.0
234	75.00	0.0	0.0
235	71.30	-17.00	0.0
236	62.80	-38.60	0.0
237	53.03	-53.03	0.0
238	38.10	-62.90	0.0
239	17.10	-71.80	0.0
240	0.0	-75.00	0.0
241	-16.90	-71.80	0.0
242	-38.50	-62.80	0.0
243	-53.03	-53.03	0.0
244	-62.70	-38.60	0.0
245	-71.30	-17.10	0.0
246	-75.00	0.0	0.0
247	-71.30	17.10	0.0
248	-62.90	38.30	0.0
249	-53.03	53.03	0.0
250	-38.60	62.80	0.0
251	-17.30	71.30	0.0
252	0.0	75.00	-40.00
253	16.80	71.60	-40.00
254	38.10	62.90	-40.00
255	53.03	53.03	-40.00
256	62.80	38.60	-40.00
257	71.30	17.00	-40.00
258	75.00	0.0	-40.00
259	71.30	-17.00	-40.00
260	62.80	-38.60	-40.00
261	53.03	-53.03	-40.00
262	38.10	-62.90	-40.00
263	17.10	-71.80	-40.00
264	0.0	-75.00	-40.00
265	-16.90	-71.80	-40.00
266	-38.50	-62.80	-40.00

267	-53.03	-53.03	-40.00
268	-62.70	-38.60	-40.00
269	-71.30	-17.10	-40.00
270	-75.00	0.0	-40.00
271	-71.30	17.10	-40.00
272	-62.90	38.30	-40.00
273	-53.03	53.03	-40.00
274	-38.60	62.80	-40.00
275	-17.30	71.30	-40.00
276	0.0	75.00	-80.00
277	16.80	71.60	-80.00
278	38.10	62.90	-80.00
279	53.03	53.03	-80.00
280	62.80	38.60	-80.00
281	71.30	17.00	-80.00
282	75.00	0.0	-80.00
283	71.30	-17.00	-80.00
284	62.80	-38.60	-80.00
285	53.03	-53.03	-80.00
286	38.10	-62.90	-80.00
287	17.10	-71.80	-80.00
288	0.0	-75.00	-80.00
289	-16.90	-71.80	-80.00
290	-38.50	-62.80	-80.00
291	-53.03	-53.03	-80.00
292	-62.70	-38.60	-80.00
293	-71.30	-17.10	-80.00
294	-75.00	0.0	-80.00
295	-71.30	17.10	-80.00
296	-62.90	38.30	-80.00
297	-53.03	53.03	-80.00
298	-38.60	62.80	-80.00
299	-17.30	71.30	-80.00
300	0.0	0.0	-110.00
301	0.0	30.00	-110.00
302	21.21	21.21	-110.00
303	30.00	0.0	-110.00
304	21.21	-21.21	-110.00
305	0.0	-30.00	-110.00
306	-21.21	-21.21	-110.00
307	-30.00	0.0	-110.00
308	-21.21	21.21	-110.00
309	0.0	60.00	-110.00
310	22.96	55.43	-110.00
311	42.43	42.43	-110.00
312	55.43	22.96	-110.00
313	60.00	0.0	-110.00
314	55.43	-22.96	-110.00

315	42.43	-42.43	-110.00
316	22.96	-55.43	-110.00
317	0.0	-60.00	-110.00
318	-22.96	-55.43	-110.00
319	-42.43	-42.43	-110.00
320	-55.43	-22.96	-110.00
321	-60.00	0.0	-110.00
322	-55.43	22.96	-110.00
323	-42.43	42.43	-110.00
324	-22.96	55.43	-110.00
325	0.0	75.00	-110.00
326	16.80	71.60	-110.00
327	38.10	62.90	-110.00
328	53.03	53.03	-110.00
329	62.80	38.60	-110.00
330	71.30	17.00	-110.00
331	75.00	0.0	-110.00
332	71.30	-17.00	-110.00
333	62.80	-38.60	-110.00
334	53.03	-53.03	-110.00
335	38.10	-62.90	-110.00
336	17.10	-71.80	-110.00
337	0.0	-75.00	-110.00
338	-16.90	-71.80	-110.00
339	-38.50	-62.80	-110.00
340	-53.03	-53.03	-110.00
341	-62.70	-38.60	-110.00
342	-71.30	-17.10	-110.00
343	-75.00	0.0	-110.00
344	-71.30	17.10	-110.00
345	-62.90	38.30	-110.00
346	-53.03	53.03	-110.00
347	-38.60	62.80	-110.00
348	-17.30	71.30	-110.00
349	0.0	90.00	-110.00
350	17.56	88.27	-110.00
351	34.44	83.15	-110.00
352	50.00	74.83	-110.00
353	63.64	63.64	-110.00
354	74.83	50.00	-110.00
355	83.15	34.44	-110.00
356	88.27	17.56	-110.00
357	90.00	0.0	-110.00
358	88.27	-17.56	-110.00
359	83.15	-34.44	-110.00
360	74.83	-50.00	-110.00
361	63.64	-63.64	-110.00
362	50.00	-74.83	-110.00

363	34.44	-83.15	-110.00
364	17.56	-88.27	-110.00
365	0.0	-90.00	-110.00
366	-17.56	-88.27	-110.00
367	-34.44	-83.15	-110.00
368	-50.00	-74.83	-110.00
369	-63.64	-63.64	-110.00
370	-74.83	-50.00	-110.00
371	-83.15	-34.44	-110.00
372	-88.27	-17.56	-110.00
373	-90.00	0.0	-110.00
374	-88.27	17.56	-110.00
375	-83.15	34.44	-110.00
376	-74.83	50.00	-110.00
377	-63.64	63.64	-110.00
378	-50.00	74.83	-110.00
379	-34.44	83.15	-110.00
380	-17.56	88.27	-110.00
381	0.0	90.00	-95.00
382	34.44	83.15	-95.00
383	63.64	63.64	-95.00
384	83.15	34.44	-95.00
385	90.00	0.0	-95.00
386	83.15	-34.44	-95.00
387	63.64	-63.64	-95.00
388	34.44	-83.15	-95.00
389	0.0	-90.00	-95.00
390	-34.44	-83.15	-95.00
391	-63.64	-63.64	-95.00
392	-83.15	-34.44	-95.00
393	-90.00	0.0	-95.00
394	-83.15	34.44	-95.00
395	-63.64	63.64	-95.00
396	-34.44	83.15	-95.00
397	0.0	90.00	-80.00
398	17.56	88.27	-80.00
399	34.44	83.15	-80.00
400	50.00	74.83	-80.00
401	63.64	63.64	-80.00
402	74.83	50.00	-80.00
403	83.15	34.44	-80.00
404	88.27	17.56	-80.00
405	90.00	0.0	-80.00
406	88.27	-17.56	-80.00
407	83.15	-34.44	-80.00
408	74.83	-50.00	-80.00
409	63.64	-63.64	-80.00
410	50.00	-74.83	-80.00

411	34.44	-83.15	-80.00
412	17.56	-88.27	-80.00
413	0.0	-90.00	-80.00
414	-17.56	-88.27	-80.00
415	-34.44	-83.15	-80.00
416	-50.00	-74.83	-80.00
417	-63.64	-63.64	-80.00
418	-74.83	-50.00	-80.00
419	-83.15	-34.44	-80.00
420	-88.27	-17.56	-80.00
421	-90.00	0.0	-80.00
422	-88.27	17.56	-80.00
423	-83.15	34.44	-80.00
424	-74.83	50.00	-80.00
425	-63.64	63.64	-80.00
426	-50.00	74.83	-80.00
427	-34.44	83.15	-80.00
428	-17.56	88.27	-80.00
429	0.0	90.00	-60.00
430	34.44	83.15	-60.00
431	63.64	63.64	-60.00
432	83.15	34.44	-60.00
433	90.00	0.0	-60.00
434	83.15	-34.44	-60.00
435	63.64	-63.64	-60.00
436	34.44	-83.15	-60.00
437	0.0	-90.00	-60.00
438	-34.44	-83.15	-60.00
439	-63.64	-63.64	-60.00
440	-83.15	-34.44	-60.00
441	-90.00	0.0	-60.00
442	-83.15	34.44	-60.00
443	-63.64	63.64	-60.00
444	-34.44	83.15	-60.00
445	0.0	90.00	-40.00
446	17.56	88.27	-40.00
447	34.44	83.15	-40.00
448	50.00	74.83	-40.00
449	63.64	63.64	-40.00
450	74.83	50.00	-40.00
451	83.15	34.44	-40.00
452	88.27	17.56	-40.00
453	90.00	0.0	-40.00
454	88.27	-17.56	-40.00
455	83.15	-34.44	-40.00
456	74.83	-50.00	-40.00
457	63.64	-63.64	-40.00
458	50.00	-74.83	-40.00

459	34.44	-83.15	-40.00
460	17.56	-88.27	-40.00
461	0.0	-90.00	-40.00
462	-17.56	-88.27	-40.00
463	-34.44	-83.15	-40.00
464	-50.00	-74.83	-40.00
465	-63.64	-63.64	-40.00
466	-74.83	-50.00	-40.00
467	-83.15	-34.44	-40.00
468	-88.27	-17.56	-40.00
469	-90.00	0.0	-40.00
470	-88.27	17.56	-40.00
471	-83.15	34.44	-40.00
472	-74.83	50.00	-40.00
473	-63.64	63.64	-40.00
474	-50.00	74.83	-40.00
475	-34.44	83.15	-40.00
476	-17.56	88.27	-40.00
477	0.0	90.00	-20.00
478	34.44	83.15	-20.00
479	63.64	63.64	-20.00
480	83.15	34.44	-20.00
481	90.00	0.0	-20.00
482	83.15	-34.44	-20.00
483	63.64	-63.64	-20.00
484	34.44	-83.15	-20.00
485	0.0	-90.00	-20.00
486	-34.44	-83.15	-20.00
487	-63.64	-63.64	-20.00
488	-83.15	-34.44	-20.00
489	-90.00	0.0	-20.00
490	-83.15	34.44	-20.00
491	-63.64	63.64	-20.00
492	-34.44	83.15	-20.00
493	0.0	90.00	0.0
494	17.56	88.27	0.0
495	34.44	83.15	0.0
496	50.00	74.83	0.0
497	63.64	63.64	0.0
498	74.83	50.00	0.0
499	83.15	34.44	0.0
500	88.27	17.56	0.0
501	90.00	0.0	0.0
502	88.27	-17.56	0.0
503	83.15	-34.44	0.0
504	74.83	-50.00	0.0
505	63.64	-63.64	0.0
506	50.00	-74.83	0.0

507	34.44	-83.15	0.0
508	17.56	-88.27	0.0
509	0.0	-90.00	0.0
510	-17.56	-88.27	0.0
511	-34.44	-83.15	0.0
512	-50.00	-74.83	0.0
513	-63.64	-63.64	0.0
514	-74.83	-50.00	0.0
515	-83.15	-34.44	0.0
516	-88.27	-17.56	0.0
517	-90.00	0.0	0.0
518	-88.27	17.56	0.0
519	-83.15	34.44	0.0
520	-74.83	50.00	0.0
521	-63.64	63.64	0.0
522	-50.00	74.83	0.0
523	-34.44	83.15	0.0
524	-17.56	88.27	0.0
525	0.0	90.00	20.00
526	34.44	83.15	20.00
527	63.64	63.64	20.00
528	83.15	34.44	20.00
529	90.00	0.0	20.00
530	83.15	-34.44	20.00
531	63.64	-63.64	20.00
532	34.44	-83.15	20.00
533	0.0	-90.00	20.00
534	-34.44	-83.15	20.00
535	-63.64	-63.64	20.00
536	-83.15	-34.44	20.00
537	-90.00	0.0	20.00
538	-83.15	34.44	20.00
539	-63.64	63.64	20.00
540	-34.44	83.15	20.00
541	0.0	90.00	40.00
542	17.56	88.27	40.00
543	34.44	83.15	40.00
544	50.00	74.83	40.00
545	63.64	63.64	40.00
546	74.83	50.00	40.00
547	83.15	34.44	40.00
548	88.27	17.56	40.00
549	90.00	0.0	40.00
550	88.27	-17.56	40.00
551	83.15	-34.44	40.00
552	74.83	-50.00	40.00
553	63.64	-63.64	40.00
554	50.00	-74.83	40.00

555	34.44	-83.15	40.00
556	17.56	-88.27	40.00
557	0.0	-90.00	40.00
558	-17.56	-88.27	40.00
559	-34.44	-83.15	40.00
560	-50.00	-74.83	40.00
561	-63.64	-63.64	40.00
562	-74.83	-50.00	40.00
563	-83.15	-34.44	40.00
564	-88.27	-17.56	40.00
565	-90.00	0.0	40.00
566	-88.27	17.56	40.00
567	-83.15	34.44	40.00
568	-74.83	50.00	40.00
569	-63.64	63.64	40.00
570	-50.00	74.83	40.00
571	-34.44	83.15	40.00
572	-17.56	88.27	40.00
573	0.0	90.00	60.00
574	34.44	83.15	60.00
575	63.64	63.64	60.00
576	83.15	34.44	60.00
577	90.00	0.0	60.00
578	83.15	-34.44	60.00
579	63.64	-63.64	60.00
580	34.44	-83.15	60.00
581	0.0	-90.00	60.00
582	-34.44	-83.15	60.00
583	-63.64	-63.64	60.00
584	-83.15	-34.44	60.00
585	-90.00	0.0	60.00
586	-83.15	34.44	60.00
587	-63.64	63.64	60.00
588	-34.44	83.15	60.00
589	0.0	90.00	80.00
590	17.56	88.27	80.00
591	34.44	83.15	80.00
592	50.00	74.83	80.00
593	63.64	63.64	80.00
594	74.83	50.00	80.00
595	83.15	34.44	80.00
596	88.27	17.56	80.00
597	90.00	0.0	80.00
598	88.27	-17.56	80.00
599	83.15	-34.44	80.00
600	74.83	-50.00	80.00
601	63.64	-63.64	80.00
602	50.00	-74.83	80.00

603	34.44	-83.15	80.00
604	17.56	-88.27	80.00
605	0.0	-90.00	80.00
606	-17.56	-88.27	80.00
607	-34.44	-83.15	80.00
608	-50.00	-74.83	80.00
609	-63.64	-63.64	80.00
610	-74.83	-50.00	80.00
611	-83.15	-34.44	80.00
612	-88.27	-17.56	80.00
613	-90.00	0.0	80.00
614	-88.27	17.56	80.00
615	-83.15	34.44	80.00
616	-74.83	50.00	80.00
617	-63.64	63.64	80.00
618	-50.00	74.83	80.00
619	-34.44	83.15	80.00
620	-17.56	88.27	80.00
621	0.0	90.00	95.00
622	34.44	83.15	95.00
623	63.64	63.64	95.00
624	83.15	34.44	95.00
625	90.00	0.0	95.00
626	83.15	-34.44	95.00
627	63.64	-63.64	95.00
628	34.44	-83.15	95.00
629	0.0	-90.00	95.00
630	-34.44	-83.15	95.00
631	-63.64	-63.64	95.00
632	-83.15	-34.44	95.00
633	-90.00	0.0	95.00
634	-83.15	34.44	95.00
635	-63.64	63.64	95.00
636	-34.44	83.15	95.00
637	0.0	90.00	110.00
638	17.56	88.27	110.00
639	34.44	83.15	110.00
640	50.00	74.83	110.00
641	63.64	63.64	110.00
642	74.83	50.00	110.00
643	83.15	34.44	110.00
644	88.27	17.56	110.00
645	90.00	0.0	110.00
646	88.27	-17.56	110.00
647	83.15	-34.44	110.00
648	74.83	-50.00	110.00
649	63.64	-63.64	110.00
650	50.00	-74.83	110.00

651	34.44	-83.15	110.00
652	17.56	-88.27	110.00
653	0.0	-90.00	110.00
654	-17.56	-88.27	110.00
655	-34.44	-83.15	110.00
656	-50.00	-74.83	110.00
657	-63.64	-63.64	110.00
658	-74.83	-50.00	110.00
659	-83.15	-34.44	110.00
660	-88.27	-17.56	110.00
661	-90.00	0.0	110.00
662	-88.27	17.56	110.00
663	-83.15	34.44	110.00
664	-74.83	50.00	110.00
665	-63.64	63.64	110.00
666	-50.00	74.83	110.00
667	-34.44	83.15	110.00
668	-17.56	88.27	110.00
669	0.0	75.00	110.00
670	16.80	71.60	110.00
671	38.10	62.90	110.00
672	53.03	53.03	110.00
673	62.80	38.60	110.00
674	71.30	17.00	110.00
675	75.00	0.0	110.00
676	71.30	-17.00	110.00
677	52.80	-38.60	110.00
678	53.03	-53.03	110.00
679	38.10	-62.90	110.00
680	17.10	-71.80	110.00
681	0.0	-75.00	110.00
682	-16.90	-71.80	110.00
683	-38.50	-62.80	110.00
684	-53.03	-53.03	110.00
685	-52.70	-38.60	110.00
686	-71.30	-17.10	110.00
687	-75.00	0.0	110.00
688	-71.30	17.10	110.00
689	-62.90	38.30	110.00
690	-53.03	53.03	110.00
691	-38.60	62.80	110.00
692	-17.30	71.30	110.00
693	0.0	60.00	110.00
694	22.96	55.43	110.00
695	42.43	42.43	110.00
696	55.43	22.96	110.00
697	60.00	0.0	110.00
698	55.43	-22.96	110.00

699	42.43	-42.43	110.00
700	22.96	-55.43	110.00
701	0.0	-60.00	110.00
702	-22.96	-55.43	110.00
703	-42.43	-42.43	110.00
704	-55.43	-22.96	110.00
705	-60.00	0.0	110.00
706	-55.43	22.96	110.00
707	-42.43	42.43	110.00
708	-22.96	55.43	110.00
709	0.0	30.00	110.00
710	21.21	21.21	110.00
711	30.00	0.0	110.00
712	21.21	-21.21	110.00
713	0.0	-30.00	110.00
714	-21.21	-21.21	110.00
715	-30.00	0.0	110.00
716	-21.21	21.21	110.00
717	0.0	0.0	110.00

```
C-----------------------------------------------------------------------
C       THIS PROGRAM DERIVES THE NAME VECTOR THROUGH THE USE OF THE
C       CONNECTIVITY MATRIX.
C-----------------------------------------------------------------------
C
C
        INTEGER*2 CONN,JA,MAME,NAME,JB
        DIMENSION CONN(128,15),JA(183),MAME(183,94),NAME(4615),JB(184)
      5 FORMAT(16I5)
      6 FORMAT(16I5)
     20 FORMAT(//2X,'ELEMENT',1X,'NODE 1',1X,'NODE 2',1X,'NODE 3',1X,'NODE
       1 4',1X,'NODE 5',1X,'NODE 6',1X,'NODE 7',1X,'NODE 8',1X,'NODE 9',1X
       1,'NODE10',1X,'NODE11',1X,'NODE12',1X,'NODE13',1X,'NODE14',1X,'NODE
       115')
     25 FORMAT(I6,15I7)
    205 FORMAT(25I5)
    204 FORMAT('0      JA(I) VECTOR')
    207 FORMAT('0      JB(I) VECTOR')
    208 FORMAT('0      NAME(JC) VECTOR')
    215 FORMAT(/2X,'JC=',I10)
C
C     THIS PROGRAM CALCULATES THE NAME ARRAY
C
        NUMNP=183
        NUMEL=128
        NELDOF=15
        DO 455 I=1,128
        READ(5,150)I,(CONN(I,J),J=1,15)
    150 FORMAT(16I5)
        WRITE(6,150)I,(CONN(I,J),J=1,15)
    455 CONTINUE
        DO 40 I=1,NUMNP
        JA(I)=1
     40 CONTINUE
        DO 60 I=1,NUMNP
        DO 50 J=1,93
        MAME(I,J)=0
     50 CONTINUE
     60 CONTINUE
        DO 65 I=1,NUMNP
        MAME(I,1)=I
     65 CONTINUE
        DO 100 I=1,NUMEL
        DO 90 J=1,NELDOF
        JJ=CONN(I,J)
        IF(JJ.GT.183) GO TO 90
        DO 80 K=1,NELDOF
        IF(K.EQ.J) GO TO 80
```

```
      KK=CONN(I,K)
      IF(KK.GT.183) GO TO 80
      JA(JJ)=JA(JJ)+1
      JAA=JA(JJ)
      DO 70 L=2,JAA
      JJL=MAME(JJ,L)
      IF(JJL.EQ.KK) JA(JJ)=JA(JJ)-1
      IF(JJL.EQ.KK) GO TO 80
      IF(JJL.EQ.0) MAME(JJ,JAA)=KK
   70 CONTINUE
   80 CONTINUE
   90 CONTINUE
  100 CONTINUE
C
      JB(1)=1
      JC=0
      DO 200 I=1,183
      JN=JA(I)
      JB(I+1)=JB(I)+JA(I)
      JC=JC+JA(I)
  200 CONTINUE
      WRITE(6,215) JC
      DO 250 I=1,183
      JAA=JA(I)
      JBL=JB(I)
      DO 240 J=1,JAA
      JJ=JBL+J-1
      NAME(JJ)=MAME(I,J)
  240 CONTINUE
  250 CONTINUE
      WRITE(6,204)
      WRITE(6,205)(JA(I),I=1,183)
      WRITE(6,207)
      WRITE(6,205)(JB(I),I=1,183)
      WRITE(6,208)
      WRITE(6,205) (NAME(I),I=1,JC)
      STOP
      END
```

```
C
C-----------------------------------------------------------------------
C      THIS IS THE MAIN PROGRAM WHICH CALCULATES THE BIGG AND BIGGG
C      SYSTEM MATRICES BY USING SUBROUTINES FLOWIE AND TANYA. THE SYSTEM
C      MATRICES ARE THEN PUT ON TAPE NUMBER NPS182.
C-----------------------------------------------------------------------
C
      REAL*8 G,GG,BIGG,BIGGG
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                 M-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN               M-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(183),NNZ,JC,NUMNP,NUMEL,NELDOF,
     1CONN(128,15)
      COMMON/GTRY1/X(505),P(505),Z(505)
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)                        M-060
      COMMON/GMAT/G(15,15)
      COMMON/GGMAT/GG(15,15)
      COMMON/SYSMT1/BIGG(4615)
      COMMON/SYSMT2/BIGGG(4615)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)                    M-100
  150 FORMAT(16I5)
  160 FORMAT(25I5)
  211 FORMAT(8E16.8)
  609 FORMAT(12F10.4)
  702 FORMAT(20I6)
  800 FORMAT(10F8.2)
  952 FORMAT(20I4)
      NNZ=183
      NUMEL=128
      NUMNP=505
      JC=4615
      NELDOF=15
      DO 455 I=1,NUMEL                                                  M-120
      READ(5,150)I,(CONN(I,J),J=1,NELDOF)                               M-125
  455 CONTINUE                                                          M-130
      READ(5,952)(JA(I),I=1,NNZ)
      READ(5,952)(JB(I),I=1,NNZ)
      READ(5,952)(NAME(I),I=1,JC)                                       M-160
      READ(5,800)(X(I),I=1,NUMNP)                                       M-140
      READ(5,800)(P(I),I=1,NUMNP)                                       M-145
      READ(5,800)(Z(I),I=1,NUMNP)
      DO 465 I=1,JC                                                     M-355
      BIGG(I)=0.0                                                       M-360
      BIGGG(I)=0.0                                                      M-365
  465 CONTINUE                                                          M-370
      CALL TANYA                                                        M-405
      CALL FLOWIE                                                       M-415
      STOP                                                              M-615
```

```
      END                                                               M-620
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE EVALUATES THE GG ELEMENT MATRIX USING 20 POINTS OF
C     INTEGRATION. THE GG ELEMENT MATRIX IS THEN INSERTED INTO THE BIGGG
C     SYSTEM MATRIX.
C-----------------------------------------------------------------------
C
      SUBROUTINE TANYA
      REAL*8 GG,BIGGG
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                 T-015
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN
      COMMON/GTRY2/NAME(8199),JA(299),JB(299),NNZ,JC,NUMNP,NUMEL,NELDOF,
     1CONN(192,15)
      COMMON/GTRY1/X(717),P(717),Z(717)
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)
      COMMON/GGMAT/GG(15,15)
      COMMON/SYSMT2/BIGGG(8199)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
      WS(1)=0.236926885056189
      WS(2)=0.478628670499366
      WS(3)=0.568888888888889
      WS(4)=0.478628670499366
      WS(5)=0.236926885056189
      S(1)=0.906179845938664
      S(2)=0.538469310105683
      S(3)=0.0
      S(4)=-.538469310105683
      S(5)=-.906179845938664
      WL(1)=-0.281250
      WL(2)=0.260416666666667
      WL(3)=0.260416666666667
      WL(4)=0.260416666666667
      CL1(1)=0.333333333333333
      CL1(2)=0.60
      CL1(3)=0.20
      CL1(4)=0.20
      CL2(1)=0.333333333333333
      CL2(2)=0.20
      CL2(3)=0.60
      CL2(4)=0.20
      CL3(1)=0.333333333333333
      CL3(2)=0.20
      CL3(3)=0.20
      CL3(4)=0.60
      DO 80 K=1,5
      DO 90 M=1,4
```

```
      IF(K.EQ.1) GO TO 10
      IF(K.EQ.2) GO TO 20
      IF(K.EQ.3) GO TO 30
      IF(K.EQ.4) GO TO 40
      IF(K.EQ.5) GO TO 50
   10 N=M
      GO TO 60
   20 N=M+4
      GO TO 60
   30 N=M+8
      GO TO 60
   40 N=M+12
      GO TO 60
   50 N=M+16
   60 CONTINUE
      FN(1,N)= 0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(2,N)=  2.0*CL1(M)*CL2(M)*(1.0+S(K))
      FN(3,N)= 0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(4,N)=  2.0*CL2(M)*CL3(M)*(1.0+S(K))
      FN(5,N)= 0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(6,N)=  2.0*CL1(M)*CL3(M)*(1.0+S(K))
      FN(7,N)= CL1(M)*(1.0-S(K)**2)
      FN(8,N)= CL2(M)*(1.0-S(K)**2)
      FN(9,N)= CL3(M)*(1.0-S(K)**2)
      FN(10,N)=0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(11,N)= 2.0*CL1(M)*CL2(M)*(1.0-S(K))
      FN(12,N)=0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(13,N)= 2.0*CL2(M)*CL3(M)*(1.0-S(K))
      FN(14,N)=0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(15,N)= 2.0*CL1(M)*CL3(M)*(1.0-S(K))
      DL1(1,N)= 0.50*((1.0+S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(2,N)=  2.0*CL2(M)*(1.0+S(K))
      DL1(3,N)=0.0
      DL1(4,N)=0.0
      DL1(5,N)=0.0
      DL1(6,N)= 2.0*CL3(M)*(1.0+S(K))
      DL1(7,N)= (1.0-S(K)**2)
      DL1(8,N)=0.0
      DL1(9,N)=0.0
      DL1(10,N)=0.50*((1.0-S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(11,N)= 2.0*CL2(M)*(1.0-S(K))
      DL1(12,N)=0.0
      DL1(13,N)=0.0
      DL1(14,N)=0.0
      DL1(15,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(1,N)=0.0
      DL2(2,N)=  2.0*CL1(M)*(1.0+S(K))
      DL2(3,N)= 0.50*((1.0+S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
```

```
      DL2(4,N)=  2.0*CL3(M)*(1.0+S(K))
      DL2(5,N)=0.0
      DL2(6,N)=0.0
      DL2(7,N)=0.0
      DL2(8,N)= (1.0-S(K)**2)
      DL2(9,N)=0.0
      DL2(10,N)=0.0
      DL2(11,N)= 2.0*CL1(M)*(1.0-S(K))
      DL2(12,N)=0.50*((1.0-S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
      DL2(13,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(14,N)=0.0
      DL2(15,N)=0.0
      DL3(1,N)=0.0
      DL3(2,N)=0.0
      DL3(3,N)=0.0
      DL3(4,N)=  2.0*CL2(M)*(1.0+S(K))
      DL3(5,N)= 0.50*((1.0+S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(6,N)=  2.0*CL1(M)*(1.0+S(K))
      DL3(7,N)=0.0
      DL3(8,N)=0.0
      DL3(9,N)= (1.0-S(K)**2)
      DL3(10,N)=0.0
      DL3(11,N)=0.0
      DL3(12,N)=0.0
      DL3(13,N)= 2.0*CL2(M)*(1.0-S(K))
      DL3(14,N)=0.50*((1.0-S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(15,N)= 2.0*CL1(M)*(1.0-S(K))
      DS(1,N)=0.50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(2,N)=  2.0*CL1(M)*CL2(M)
      DS(3,N)=0.50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(4,N)=  2.0*CL2(M)*CL3(M)
      DS(5,N)=0.50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(6,N)=  2.0*CL1(M)*CL3(M)
      DS(7,N)= -2.0*CL1(M)*S(K)
      DS(8,N)= -2.0*CL2(M)*S(K)
      DS(9,N)= -2.0*CL3(M)*S(K)
      DS(10,N)=-.50*(2.*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(11,N)= -2.0*CL1(M)*CL2(M)
      DS(12,N)=-.50*(2.*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(13,N)= -2.0*CL2(M)*CL3(M)
      DS(14,N)=-.50*(2.*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(15,N)= -2.0*CL1(M)*CL3(M)
   90 CONTINUE
   80 CONTINUE
      DO 510 L=1,NUMEL
      DO 475 I=1,NELDOF
      DO 470 J=1,NELDOF
      GG(I,J)=0.0
```

```
470 CONTINUE
475 CONTINUE
    DO 480 JJ=1,NELDOF
    NN=CONN(L,JJ)
    XX(JJ)=X(NN)
    YY(JJ)=P(NN)
    ZZ(JJ)=Z(NN)
480 CONTINUE
    A5=2.0/(ZZ(1)-ZZ(15))
    DETJJ=0.0
    DO 200 K=1,5
    DETJ1=0.0
    DO 210 M=1,4
    IF(K.EQ.1) GO TO 11
    IF(K.EQ.2) GO TO 21
    IF(K.EQ.3) GO TO 31
    IF(K.EQ.4) GO TO 41
    IF(K.EQ.5) GO TO 51
 11 N=M
    GO TO 61
 21 N=M+4
    GO TO 61
 31 N=M+8
    GO TO 61
 41 N=M+12
    GO TO 61
 51 N=M+16
 61 CONTINUE
    T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
    T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)
    T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
    T4=(DL1(15,N)-DL3(15,N))*XX(15)
    DJ11=T1+T2+T3+T4
    D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
    D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
    D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
    D4=(DL1(15,N)-DL3(15,N))*YY(15)
    DJ12=D1+D2+D3+D4
    T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
    T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)
    T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
    T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
    DJ21=T5+T6+T7+T8
    D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
    D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)
    D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
    D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
    DJ22=D5+D6+D7+D8
```

```
      DETJ(N)=DJ11*DJ22-DJ12*DJ21
      DETJ1=DETJ1+DETJ(N)*WL(M)
  210 CONTINUE
      DETJJ=DETJJ+WS(K)*DETJ1
  200 CONTINUE
      DO 600 J=1,15
      DO 610 I=1,15
      FGG=0.0
      DO 660 K=1,5
      FFG=0.0
      DO 670 M=1,4
      IF(K.EQ.1) GO TO 12
      IF(K.EQ.2) GO TO 22
      IF(K.EQ.3) GO TO 32
      IF(K.EQ.4) GO TO 42
      IF(K.EQ.5) GO TO 52
   12 N=M
      GO TO 62
   22 N=M+4
      GO TO 62
   32 N=M+8
      GO TO 62
   42 N=M+12
      GO TO 62
   52 N=M+16
   62 CONTINUE
      DXDL5=DL1(1,N)*XX(1)+(DL1(2,N)-DL2(2,N))*XX(2)-DL2(3,N)*XX(3)
      DXDL6=-(DL2(4,N)+DL3(4,N))*XX(4)-DL3(5,N)*XX(5)
      DXDL7=(DL1(6,N)-DL3(6,N))*XX(6)+DL1(7,N)*XX(7)-DL2(8,N)*XX(8)
      DXDL8=-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+(DL1(11,N)-DL2(11,N))*XX(11
     1)
      DXDL9=-DL2(12,N)*XX(12)-(DL2(13,N)+DL3(13,N))*XX(13)-DL3(14,N)*XX(
     114)+(DL1(15,N)-DL3(15,N))*XX(15)
      DXDL1=DXDL5+DXDL6+DXDL7+DXDL8+DXDL9
      DYDL5=DL1(1,N)*YY(1)+(DL1(2,N)-DL2(2,N))*YY(2)-DL2(3,N)*YY(3)
      DYDL6=-(DL2(4,N)+DL3(4,N))*YY(4)-DL3(5,N)*YY(5)
      DYDL7=(DL1(6,N)-DL3(6,N))*YY(6)+DL1(7,N)*YY(7)-DL2(8,N)*YY(8)
      DYDL8=-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+(DL1(11,N)-DL2(11,N))*YY(11
     1)
      DYDL9=-DL2(12,N)*YY(12)-(DL2(13,N)+DL3(13,N))*YY(13)-DL3(14,N)*YY(
     114)+(DL1(15,N)-DL3(15,N))*YY(15)
      DYDL1=DYDL5+DYDL6+DYDL7+DYDL8+DYDL9
      DXDL55=-DL1(1,N)*XX(1)+(DL2(2,N)-DL1(2,N))*XX(2)+DL2(3,N)*XX(3)
      DXDL66=(DL2(4,N)-DL3(4,N))*XX(4)-DL3(5,N)*XX(5)-DL1(7,N)*XX(7)
      DXDL77=-(DL1(6,N)+DL3(6,N))*XX(6)+DL2(8,N)*XX(8)-DL3(9,N)*XX(9)
      DXDL88=-DL1(10,N)*XX(10)+(DL2(11,N)-DL1(11,N))*XX(11)
      DXDL99=DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)-DL3(14,N)*XX(
     114)-(DL1(15,N)+DL3(15,N))*XX(15)
```

```
      DXDL2=DXDL55+DXDL66+DXDL77+DXDL88+DXDL99
      DYDL55=-DL1(1,N)*YY(1)+(DL2(2,N)-DL1(2,N))*YY(2)+DL2(3,N)*YY(3)
      DYDL66=(DL2(4,N)-DL3(4,N))*YY(4)-DL3(5,N)*YY(5)-DL1(7,N)*YY(7)
      DYDL77=-(DL1(6,N)+DL3(6,N))*YY(6)+DL2(8,N)*YY(8)-DL3(9,N)*YY(9)
      DYDL88=-DL1(10,N)*YY(10)+(DL2(11,N)-DL1(11,N))*YY(11)
      DYDL99=DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)-DL3(14,N)*YY(
     114)-(DL1(15,N)+DL3(15,N))*YY(15)
      DYDL2=DYDL55+DYDL66+DYDL77+DYDL88+DYDL99
      DL1DY=1.0/DYDL1
      DL2DX=1.0/DXDL2
      DL1DX=1.0/DXDL1
      DL2DY=1.0/DYDL2
      BB11=(DL1DX**2)+(DL1DY**2)
      BB12=DL1DX*DL2DX+DL1DY*DL2DY
      BB22=(DL2DX**2)+(DL2DY**2)
      H1=BB11*((DL1(J,N)-DL3(J,N))*(DL1(I,N)-DL3(I,N)))
      H2=BB12*((DL2(J,N)-DL3(J,N))*(DL1(I,N)-DL3(I,N)))
      H3=BB12*((DL1(J,N)-DL3(J,N))*(DL2(I,N)-DL3(I,N)))
      H4=BB22*((DL2(J,N)-DL3(J,N))*(DL2(I,N)-DL3(I,N)))
      H5=(A5**2)*DS(J,N)*DS(I,N)
      H6=(H1+H2+H3+H4+H5)*DETJ(N)
      FFG=FFG+WL(M)*H6
  670 CONTINUE
      FGG=WS(K)*FFG+FGG
  660 CONTINUE
      GG(J,I)=FGG/A5
  610 CONTINUE
  600 CONTINUE
C     *** INSERT ELEMENT MATRIX INTO SYSTEM MATRIX *****
      DO 505 K=1,NELDOF
      KK=CONN(L,K)
      IF(KK.GT.NNZ) GO TO 505
      KKK=JA(KK)
      LLL=JB(KK)-1
      DO 500 I=1,NELDOF
      II=CONN(L,I)
      IF(II.GT.NNZ) GO TO 500
      DO 490 M=1,KKK
      MM=LLL+M
      KKM=NAME(MM)
      IF(II.EQ.KKM) GO TO 495
  490 CONTINUE
  495 CONTINUE
      BIGGG(MM)=BIGGG(MM)+GG(K,I)
  500 CONTINUE
  505 CONTINUE
  510 CONTINUE
      RETURN
```

```
      END
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE EVALJATES THE G ELEMENT MATRIX USING  21 POINTS OF
C     INTEGRATION. THE G ELEMENT MATRIX IS THEN INSERTED INTO THE BIGG
C     SYSTEM MATRIX.
C-----------------------------------------------------------------------
C
      SUBROUTINE FLOWIE
      REAL*8 G,BIGG
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ
      INTEGER*2 NAME,JA,JB,NNZ,JC,NJMNP,NUMEL,NELDOF,CONN
      COMMON/GTRY2/NAME(8199),JA(299),JB(299),NNZ,JC,NUMNP,NUMEL,NELDOF,
     1CONN(192,15)
      COMMON/GTRY1/X(717),P(717),Z(717)
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)
      COMMON/GMAT/G(15,15)
      COMMON/SYSMT1/BIGG(8199)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
      WS(1)=0.555555555555556
      WS(2)=0.888888888888889
      WS(3)=0.555555555555556
      S(1)=0.774596669241483
      S(2)=0.0
      S(3)=-.774596669241483
      WL(1)=0.112500
      WL(2)=0.066197J7500
      WL(3)=0.06619707500
      WL(4)=0.06619707500
      WL(5)=J.0629695900
      WL(6)=0.0629695900
      WL(7)=0.0629695900
      CL1(1)=0.333333333333333
      CL1(2)=0.059615870
      CL1(3)=0.47J142060
      CL1(4)=0.470142060
      CL1(5)=0.797426990
      CL1(5)=0.101286510
      CL1(7)=0.101286510
      CL2(1)=0.333333333333333
      CL2(2)=0.470142060
      CL2(3)=0.059615870
      CL2(4)=0.470142060
      CL2(5)=0.101286510
      CL2(6)=0.797426990
      CL2(7)=0.101286510
      CL3(1)=0.333333333333333
```

```
      CL3(2)=0.470142060
      CL3(3)=0.470142060
      CL3(4)=0.059615870
      CL3(5)=0.101286510
      CL3(6)=0.101286510
      CL3(7)=0.797426990
      DO 10 K=1,3
      DO 20 M=1,7
C   THE 3 X 7 MATRICES ARE BEING CONVERTED TO 1 X 21 VECTORS
      IF(K-2) 30,40,50
   30 N=M
      GO TO 60
   40 N=M+7
      GO TO 60
   50 N=M+14
   60 CONTINUE
      FN(1,N)= 0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(2,N)=  2.0*CL1(M)*CL2(M)*(1.0+S(K))
      FN(3,N)= 0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(4,N)=  2.0*CL2(M)*CL3(M)*(1.0+S(K))
      FN(5,N)= 0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(6,N)=  2.0*CL1(M)*CL3(M)*(1.0+S(K))
      FN(7,N)= CL1(M)*(1.0-S(K)**2)
      FN(8,N)= CL2(M)*(1.0-S(K)**2)
      FN(9,N)= CL3(M)*(1.0-S(K)**2)
      FN(10,N)=0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(11,N)= 2.0*CL1(M)*CL2(M)*(1.0-S(K))
      FN(12,N)=0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(13,N)= 2.0*CL2(M)*CL3(M)*(1.0-S(K))
      FN(14,N)=0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(15,N)= 2.0*CL1(M)*CL3(M)*(1.0-S(K))
      DL1(1,N)= 0.50*((1.0+S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(2,N)=   2.0*CL2(M)*(1.0+S(K))
      DL1(3,N)=0.0
      DL1(4,N)=0.0
      DL1(5,N)=0.0
      DL1(6,N)= 2.0*CL3(M)*(1.0+S(K))
      DL1(7,N)= (1.0-S(K)**2)
      DL1(8,N)=0.0
      DL1(9,N)=0.0
      DL1(10,N)=0.50*((1.0-S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(11,N)= 2.0*CL2(M)*(1.0-S(K))
      DL1(12,N)=0.0
      DL1(13,N)=0.0
      DL1(14,N)=0.0
      DL1(15,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(1,N)=0.0
      DL2(2,N)=   2.0*CL1(M)*(1.0+S(K))
```

```
      DL2(3,N)= 0.50*((1.0+S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
      DL2(4,N)=  2.0*CL3(M)*(1.0+S(K))
      DL2(5,N)=0.0
      DL2(6,N)=0.0
      DL2(7,N)=0.0
      DL2(8,N)= (1.0-S(K)**2)
      DL2(9,N)=0.0
      DL2(10,N)=0.0
      DL2(11,N)= 2.0*CL1(M)*(1.0-S(K))
      DL2(12,N)=0.50*((1.0-S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
      DL2(13,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(14,N)=0.0
      DL2(15,N)=0.0
      DL3(1,N)=0.0
      DL3(2,N)=0.0
      DL3(3,N)=0.0
      DL3(4,N)=  2.0*CL2(M)*(1.0+S(K))
      DL3(5,N)= 0.50*((1.0+S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(6,N)=  2.0*CL1(M)*(1.0+S(K))
      DL3(7,N)=0.0
      DL3(8,N)=0.0
      DL3(9,N)= (1.0-S(K)**2)
      DL3(10,N)=0.0
      DL3(11,N)=0.0
      DL3(12,N)=0.0
      DL3(13,N)= 2.0*CL2(M)*(1.0-S(K))
      DL3(14,N)=0.50*((1.0-S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(15,N)= 2.0*CL1(M)*(1.0-S(K))
      DS(1,N)=0.50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(2,N)=  2.0*CL1(M)*CL2(M)
      DS(3,N)=0.50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(4,N)=  2.0*CL2(M)*CL3(M)
      DS(5,N)=0.50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(6,N)=  2.0*CL1(M)*CL3(M)
      DS(7,N)= -2.0*CL1(M)*S(K)
      DS(8,N)= -2.0*CL2(M)*S(K)
      DS(9,N)= -2.0*CL3(M)*S(K)
      DS(10,N)=-.50*(2.*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(11,N)= -2.0*CL1(M)*CL2(M)
      DS(12,N)=-.50*(2.*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(13,N)= -2.0*CL2(M)*CL3(M)
      DS(14,N)=-.50*(2.*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(15,N)= -2.0*CL1(M)*CL3(M)
   20 CONTINUE
   10 CONTINUE
      DO 510 L=1,NUMEL
      DO 475 I=1,NELDOF
      DO 470 J=1,NELDOF
```

```
      G(I,J)=0.0
  470 CONTINUE
  475 CONTINUE
      DO 480  JJ=1,NELDOF
      NN=CONN(L,JJ)
      XX(JJ)=X(NN)
      YY(JJ)=P(NN)
      ZZ(JJ)=Z(NN)
  480 CONTINUE
      A5=2.0/(ZZ(1)-ZZ(15))
      DETJJ=0.0
      DO 200  K=1,3
      DETJ1=0.0
      DO 210  M=1,7
      IF(K-2)31,41,51
   31 N=M
      GO TO 61
   41 N=M+7
      GO TO 61
   51 N=M+14
   61 CONTINUE
      T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
      T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)
      T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
      T4=(DL1(15,N)-DL3(15,N))*XX(15)
      DJ11=T1+T2+T3+T4
      D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
      D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
      D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
      D4=(DL1(15,N)-DL3(15,N))*YY(15)
      DJ12=D1+D2+D3+D4
      T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
      T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)
      T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
      T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
      DJ21=T5+T6+T7+T8
      D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
      D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)
      D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
      D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
      DJ22=D5+D6+D7+D8
      DETJ(N)=DJ11*DJ22-DJ12*DJ21
      DETJ1=DETJ(N)*WL(M)+DETJ1
  210 CONTINUE
      DETJJ=DETJJ+WS(K)*DETJ1
  200 CONTINUE
      DO 600  J=1,15
      DO 610  I=1,15
```

```
      FG=0.0
      DO 660 K=1,3
      F=0.0
      DO 670 M=1,7
      IF(K-2)620,630,640
  620 N=M
      GO TO 650
  630 N=M+7
      GO TO 650
  640 N=M+14
  650 CONTINUE
      F=F+WL(M)*FN(J,N)*FN(I,N)*DETJ(N)
  670 CONTINUE
      FG=FG+WS(K)*F
  660 CONTINUE
      G(J,I)=FG/A5
  610 CONTINUE
  600 CONTINUE
C     *** INSERT ELEMENT MATRIX INTO SYSTEM MATRIX *****
      DO 505 K=1,NELDOF
      KK=CONN(L,K)
      IF(KK.GT.NNZ) GO TO 505
      KKK=JA(KK)
      LLL=JB(KK)-1
      DO 500 I=1,NELDOF
      II=CONN(L,I)
      IF(II.GT.NNZ) GO TO 500
      DO 490 M=1,KKK
      MM=LLL+M
      KKM=NAME(MM)
      IF(II.EQ.KKM) GO TO 495
  490 CONTINUE
  495 CONTINUE
      BIGG(MM)=BIGG(MM)+G(K,I)
  500 CONTINUE
  505 CONTINUE
  510 CONTINUE
      RETURN
      END
```

```
C
C-----------------------------------------------------------------------
C      THIS IS THE MAIN PROGRAM FOR THE SOLUTION OF THE FIELD EQUATIONS.
C      THE DATA FROM THE TAPE IS READ OUT, NODAL CONSTANTS ARE EVALUATED,
C      THE INTEGRATION PACKAGE IS INITIATED,ETC.
C-----------------------------------------------------------------------
C
C
C
      REAL*8 SIGF,SIGA,C1,C2,C4,C5
      REAL*8 GGG,BIGG,BIGGG,BIGH
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                M-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN              M-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,  M-020
     1CONN(128,15)                                                     M-025
      COMMON/GTRY1/X(505),P(505),Z(505)                                M-055
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)                       M-060
      DIMENSION W(7000),Y(7,183)
      COMMON/DELAY/SUM(183),PSI(93)
      COMMON/TEMP/C3(183),C6(15)
      DIMENSION SIGF(93),SIGA(183)
      COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
      COMMON/GGGMAT/GGG(15,15)                                         M-075
      COMMON/SYSMT1/BIGG(4615)                                         M-080
      COMMON/SYSMT2/BIGGG(4615)                                        M-085
      COMMON/SYSMT3/BIGH(4615)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1  M-095
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)                   M-100
      NNZ=183
      NUMEL=128
      NUMNP=505
      NELDOF=15
      JC=4615                                                          M-105
      FMAX=1.0E10
      DO 771 I=1,NUMEL
      READ(4) (CONN(I,J),J=1,NELDOF)
  771 CONTINUE
      READ(4) (JA(I),I=1,NNZ)
      READ(4) (JB(I),I=1,NNZ)
      READ(4) (NAME(I),I=1,JC)
      READ(4) (X(I),I=1,NUMNP)
      READ(4) (P(I),I=1,NUMNP)
      READ(4) (Z(I),I=1,NUMNP)
      READ(4) (BIGG(I),I=1,JC)
      READ(4) (BIGGG(I),I=1,JC)
      DO 772 I=1,3
      READ(4) WS(I),S(I)
  772 CONTINUE
```

```
      DO 773 I=1,7
      READ(4) WL(I),CL1(I),CL2(I),CL3(I)
  773 CONTINUE
      DO 781 I=1,NELDOF
      DO 782 J=1,21
      READ(4) FN(I,J),DL1(I,J),DL2(I,J),DL3(I,J),DS(I,J)
  782 CONTINUE
  781 CONTINUE
      DO 460 I=1,NNZ                                                M-520
      Y(1,I)=FMAX
      Y(2,I)=0.0                                                    M-535
      SUM(I)=0.0                                                    M-545
      IF(I.GT.93) GO TO 460                                         M-540
      PSI(I)=FMAX
  460 CONTINUE                                                      M-550
      WRITE(6,3)
    3 FORMAT('       SIGF(I)=0.0057450,   SIGA(I)=0.014010,0.0080   ')
      V=4.800E07
      Q=0.4349710
      BETA=0.006420
      B=-0.00400
      DO 445 I=1,NNZ
      IF(I.GT.93) GO TO 420
      D=0.9130
      SIGF(I)=0.0057450
      SIGA(I)=0.014010
      ZNU=2.54
      C1(I)=V*D
      C2(I)=V*SIGA(I)-V*(1.0-BETA)*SIGF(I)*ZNU
      C3(I)=(2.0179325E-13)*SIGF(I)
      C4(I)=-V*(1.0-BETA)*SIGA(I)*B
      C5(I)=-V*BETA*Q*ZNU*SIGF(I)
      GO TO 445
  420 D=1.200
      SIGA(I)=0.00800
      C1(I)=V*D
      C2(I)=V*SIGA(I)
      C3(I)=0.0
      C4(I)=0.0
      C5(I)=0.0
  445 CONTINUE
      TEND=0.0010
      NY=NNZ                                                        M-565
      NL=0                                                          M-570
      H=1.0E-19                                                     M-575
      T=0.0                                                         M-580
      HMIN=1.0E-20                                                  M-585
      HMAX=0.10                                                     M-590
```

```
      JSKF=0
      CALL SDESOL(Y,YL,T,TEND,NY,NL,NY,JSKF,5,1,H,HMIN,HMAX,1.E-2,W)
      STOP
      END
C
      SUBROUTINE SDESOL (Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,
     1HMAX,RMSEPS,W)
C
C     ------------------------------------------------------------------
C
C     SUBROUTINE SDESOL IS A DRIVER ROUTINE FOR SUBROUTINE LDASUB.
C     ITS PURPOSE IS TO SET UP THE NECESSARY REFERENCES TO A LARGE
C     BLOCK OF AUXILLARY STORAGE, AND  OBTAIN INITIAL VALUES OF
C     DERIVATIVES.
C     THE CALLING SEQUENCE FOR SDESOL IS
C
C CALL  SDESOL(Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,HMAX,RMSEPS,W)
C
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.
C
C        Y         - ARRAY DIMENSIONED (7,NY).  THIS ARRAY CONTAINS THE
C                    DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.
C                    Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VAR
C                    IABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT
C                    STEPSIZE.  ON FIRST ENTRY THE CALLER SUPPLIES THE
C                    INITIAL VALUES OF EACH VARIABLE IN Y(1,I).  ON SUB-
C                    SEQUENT ENTRIES IT IS ASSUMED THE ARRAY HAS NOT
C                    BEEN CHANGED.  TO INTERPOLATE TO NON-MESH POINTS,
C                    THESE VALUES CAN BE USED AS FOLLOWS.  IF H IS THE
C                    CURRENT STEPSIZE AND VALUES AT TIME  T+E  ARE
C                    NEEDED, LET  S = E/H  AND THEN
C
C                                                  JS
C                          I-TH VARIABLE AT  T+E IS  SUM Y(J+1,I)*S**J
C                                                  J=0
C
C                    THE VALUE OF JS IS OBTAINED IN THE CALLING PROGRAM
C                    BY  JS = IABS(JSKF/10)
C        YL        - ARRAY OF NL VARIABLES WHICH APPEAR LINEARLY.
C        T         - CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)
C        TEND      - END TIME
C        NY        - NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR
C                    VARIABLES.
C        NL        - NUMBER OF LINEAR VARIABLES
C        M         - NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST
C        JSKF      - AN INDICATOR USED BOTH ON INPUT AND OUTPUT
C                    ON INPUT, JSKF = -1 INDICATES A RESTART CALL TO
C                    SDESOL.  JSKF = 0 INDICATES AN INITIAL CALL TO
```

```
C                       SDESOL.  JSKF > 0 INDICATES A CONTINUATION OF THE
C                       PREVIOUS CALL TO SDESOL.  JSKF < -1 MAY HAVE RESULTED
C                       FROM THE USER NEGLECTING TO TEST FOR ERROR RETURNS
C                       FROM SDESOL.  BECAUSE OF THIS POSSIBILITY, JSKF < -1
C                       RESULTS IN TERMINATION OF THE RUN WITH THE
C                       APPROPRIATE COMMENT.
C                       ON OUTPUT, JSKF CONSISTS OF TWO DIGITS AND SIGN,
C                       + OR - QP.  Q IS THE ORDER OF THE FORMULA CURRENTLY
C                       BEING USED.  P INDICATES THE TYPE OF RETURN, AS
C                       FOLLOWS.
C                       JSKF > 0, P = 1 IS THE NORMAL RETURN
C                       JSKF < 0 IS AN ERROR RETURN, WITH THE FOLLOWING
C                       MEANINGS.
C                           P = 1   ERROR TEST FAILURE FOR H > HMIN
C                           P = 3   CORRECTOR FAILED TO CONVERGE FOR H > HMIN
C                           P = 4   CORRECTOR FAILED TO CONVERGE FOR FIRST
C                                   ORDER METHOD
C                           P = 5   ERROR RETURN FROM SUBROUTINE NUITSL
C                           P = 6   ERROR RETURN FROM SUBROUTINE DERVAL
C               MAXDER  - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN
C                       METHOD.  IT MUST BE NO GREATER THAN SIX.
C               IPRT    - INTERNAL PRINT CONTROL INDICATOR FOR LDASUB.
C                           IPRT = 0  NO PRINT
C                           IPRT > 0  PRINT COUNTERS, STEPSIZE, CURRENT TIME
C                                     AND VALUES OF DEPENDENT VARIABLES AT
C                                     EACH STEP.
C               H       - CURRENT STEPSIZE.  AN INITIAL VALUE MUST BE SUPPLIED
C                       BUT NEED NOT BE THE ONE WHICH MUST BE USED, SINCE THE
C                       SUBROUTINE WILL CHOSE A SMALLER ONE IF NECCESSARY TO
C                       KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED
C                       VALUE.  IT IS BETTER TO UNDERESTIMATE THE INITIAL
C                       STEPSIZE THAN TO OVERESTIMATE IT.  THE STEPSIZE IS
C                       NORMALLY NOT CHANGED BY THE USER.
C               HMIN    - MINIMUM STEPSIZE ALLOWED
C               HMAX    - MAXIMUM STEPSIZE ALLOWED
C               RMSEPS  - THE ERROR TEST CONSTANT.  THE ROOT-MEAN-SQUARE OF
C                       THE SINGLE STEP ERROR ESTIMATES,  ER(I), DIVIDED BY
C                       YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I) ) MUST BE
C                       LESS THAN EPS.  THE STEPSIZE AND/OR THE ORDER
C                       ARE VARIED TO ACHIEVE THIS.
C               W       - SCRATCH STORAGE ARRAY.  MUST BE AT LEAST 13*NY + 5*NL
C                       LOCATIONS, PLUS THOSE REQUIRED FOR STORAGE OF THE
C                       MATRIX  PW (SEE DESCRIPTION OF SUBROUTINE JACMAT).
C                       THE STORAGE OF PW WILL NORMALLY REQUIRE NO MORE THAN
C                       N**2 + 2*N LOCATIONS, AND IF COMPACT STORAGE TECH-
C                       NIQUES ARE USED, CAN BE MUCH FEWER.
C
C     ------------------------------------------------------------------
```

```
      DIMENSION Y(7,1), YL(1), W(1)
      IF (JSKF.GT.0) GO TO 120
      IF (JSKF.LT.-1) GO TO 140
      N = NY+NL
      IF (JSKF.LT.0) GO TO 110
C
C     IF THIS IS THE FIRST ENTRY, OBTAIN VALUES OF THE DERIVATIVES.
      CALL DERVAL (Y,YL,T,N,NY,W,KRETR)
      IF (KRETR.NE.0) GO TO 130
C
C     NOW SET UP STORAGE BLOCKS IN THE W ARRAY.  THIS NEEDS TO BE DONE
C     ONLY INITIALLY AND ON RESTARTS.
C
C        THE ARRAY   SAVE    STARTS AT LOCATION  1     IN THE W ARRAY
C        THE ARRAY   YLSV    STARTS AT LOCATION  NSVL  IN THE W ARRAY
C        THE ARRAY   YMAX    STARTS AT LOCATION  NYMAX IN THE W ARRAY
C        THE ARRAY   ER      STARTS AT LOCATION  NER   IN THE W ARRAY
C        THE ARRAY   ESV     STARTS AT LOCATION  NESV  IN THE W ARRAY
C        THE ARRAY   F1      STARTS AT LOCATION  NF1   IN THE W ARRAY
C        THE ARRAY   DY      STARTS AT LOCATION  NDY   IN THE W ARRAY
C        THE MATRIX  PW      STARTS AT LOCATION  NPW   IN THE W ARRAY
C
  110 NSVL = 7*NY+1
      NYMAX = NSVL+NL
      NER = NYMAX+NY
      NESV = NER+NY
      NF1 = NESV+NY
      NDY = NF1+N
      NPW = NDY+N
  120 JS = JSKF
      CALL LDASUB (Y,YL,T,TEND,N,NY,M,JS,KF,MAXDER,IPRT,H,HMIN,HMAX,
     1RMSEPS,W,W(NSVL),W(NYMAX),W(NER),W(NESV),W(NF1),W(NDY),W(NPW))
C
C     CODE JSKF ON RETURN FROM LDASUB
C
      JSKF = ISIGN(JS*10+IABS(KF),KF)
      RETURN
  130 JSKF = -6
      RETURN
  140 PRINT 1, JSKF
      STOP
C
C
    1 FORMAT ('0IT IS AN ERROR TO ENTER SDESOL WITH JSKF = ',I10//
     1 '    RUN HAS BEEN TERMINATED.')
      END
C
      SUBROUTINE LDASUB (Y,YL,T,TEND,N,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,
```

```
     1HMIN,HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,F1,DY,PW)
C
C     SUBROUTINE LDASUB IS A MODIFICATION OF SUBROUTINE DFASUB
C     WHICH IS DUE TO R. L. BROWN AND C. W. GEAR.  DFASUB IS DOCUMENTED
C     IN THE REPORT
C        DOCUMENTATION FOR DFASUB--
C        BY R. L. BROWN AND C. W. GEAR
C        REPORT UIUCDCS-R-73-575, JULY 1973
C        UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
C        URBANA, ILLINOIS  61801
C     THIS REPORT IS AVAILABLE FROM THE NATIONAL TECHNICAL INFORMATION
C     SERVICE OF THE U. S. DEPARTMENT OF COMMERCE UNDER ACCESSION NUMBER
C     COO-1469-225.
C
C     THE MODIFICATION HERE IS DOCUMENTED IN THE REPORT
C        A PROGRAM FOR THE NUMERICAL SOLUTION OF LARGE SPARSE SYSTEMS OF
C        ALGEBRAIC AND IMPLICITLY DEFINED STIFF DIFFERENTIAL EQUATIONS
C        BY RICHARD FRANKE
C        REPORT NPS53FE76051, MAY 1976
C        NAVAL POSTGRADUATE SCHOOL
C        MONTEREY, CALIFORNIA  93940
C
C     -------------------------------------------------------------------
C
C     THE CALLING SEQUENCE FOR LDASUB IS
C
C     CALL LDASUB(Y,YL,T,TEND,N,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,HMIN,
C     HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,F1,DY,PW)
C
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.
C        Y        - ARRAY DIMENSIONED (7,NY).  THIS ARRAY CONTAINS THE
C                   DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.
C                   Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VAR
C                   IABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT
C                   STEPSIZE.  ON FIRST ENTRY THE CALLER SUPPLIES THE
C                   INITIAL VALUES OF EACH VARIABLE IN Y(1,I) AND AN
C                   ESTIMATE OF THE INITIAL VALUES OF THE DERIVATIVES
C                   IN Y(2,I).  ON SUBSEQUENT ENTRIES IT IS ASSUMED THAT
C                   THE ARRAY HAS NOT BEEN CHANGED.  TO INTERPOLATE TO
C                   NON-MESH POINTS, THESE VALUES CAN BE USED AS FOLLOWS.
C                   IF H IS THE CURRENT STEPSIZE AND VALUES AT TIME T+E
C                   NEEDED, LET S = E/H  AND THEN
C
C                                                  NQ
C                          I-TH VARIABLE AT  T+E IS  SUM Y(J+1,I)*S**J
C                                                  J=0
C
C                   THE VALUE OF NQ IS OBTAINED IN THE CALLING PROGRAM
```

```
C                BY  NQ = JSTART.                                         LDA
C                                                                         LDA
C     YL       - ARRAY OF NL = N - NY VARIABLES WHICH APPEAR LINEARLY.LDA
C                THE USER SUPPLIES INITIAL VALUES FOR THESE VARIABLES.LDA
C     T        - CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)      LDA
C     TEND     - END TIME                                               LDA
C     N        - TOTAL NUMBER OF VARIABLES                              LDA
C     NY       - NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR         LDA
C                VARIABLES.                                             LDA
C     M        - NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST.        LDA
C                THIS NUMBER CAN BE NO GREATER THAN NY.  IF IT IS       LDA
C                GREATER THAN NY, NY VARIABLES ARE USED IN THE ERROR    LDA
C                TEST.                                                  LDA
C     JSTART   - INPUT AND OUTPUT INDICATOR.                            LDA
C                ON INPUT JSTART HAS THE FOLLOWING MEANINGS.            LDA
C                       <0  THIS INDICATES A RE-START FROM A PREVIOUS   LDA
C                           POINT FOLLOWING TERMINATION OF THE RUN OR   LDA
C                           SOLUTION OF ANOTHER PROBLEM DURING THE SAME LDA
C                           RUN.  PARAMETERS IN THE CALLING SEQUENCE    LDA
C                           MUST HAVE BEEN PRESERVED FROM THE PREVIOUS  LDA
C                           USE, PARTICULARLY THE ARRAYS                LDA
C                               SAVE, YLSV, ESV, AND PW.                LDA
C                           THESE ARRAYS MUST BE SAVED AFTER A CALL     LDA
C                           TO SUBROUTINE LDASAV, WHICH ALSO SAVES      LDA
C                           NECESSARY PARAMETERS INTERNAL TO LDASUB.    LDA
C                       =0  INDICATES AN INITIAL CALL TO LDASUB.  THE   LDA
C                           ROUTINE INITIALIZES ITSELF, SCALES THE      LDA
C                           DERIVATIVES IN Y(2,I) AND THEN PERFORMS THE LDA
C                           INTEGRATION UNTIL T > TEND.                 LDA
C                       >0  INDICATES THE SOLUTION IS TO BE CONTINUED.  LDA
C                           AFTER THE INITIAL ENTRY IT IS NEITHER       LDA
C                           DESIRABLE NOR NECESSARY TO RE-ENTER WITH    LDA
C                           JSTART = 0, SINCE THIS RE-INITIALIZES       LDA
C                           THE CODE, BEGINNING WITH A FIRST ORDER      LDA
C                           METHOD AGAIN.                               LDA
C                ON OUTPUT, JSTART IS SET TO THE VALUE OF NQ, THE       LDA
C                ORDER OF THE FORMULA CURRENTLY BEING USED.             LDA
C     KFLAG    - THE COMPLETION CODE INDICATOR, WITH THE FOLLOWING      LDA
C                MEANINGS                                               LDA
C                       +1  THE INTEGRATION WAS SUCCESSFUL              LDA
C                       -2  ERROR TEST FAILURE FOR H > HMIN             LDA
C                       -3  CORRECTOR FAILED TO CONVERGE FOR H > HMIN   LDA
C                       -4  CORRECTOR FAILED TO CONVERGE FOR FIRST      LDA
C                           ORDER METHOD                                LDA
C                       -5  ERROR RETURN FROM SUBROUTINE NUITSL         LDA
C     MAXOR    - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN THE    LDA
C                METHOD.  IT MUST BE NO GREATER THAN SIX.  IF IT IS     LDA
C                GREATER THAN SIX, THE MAXIMUM ORDER USED WILL BE SIX.LDA
```

```
C        IPRT     - INTERNAL PRINT CONTROL INDICATOR                        LDA  980
C                         = 0   NO PRINT                                    LDA  990
C                         > 0   PRINT COUNTERS, STEPSIZE, CURRENT TIME      LDA 1000
C                               AND VALUES OF DEPENDENT VARIABLES AT        LDA 1010
C                               EACH STEP.                                  LDA 1020
C        H        - CURRENT STEPSIZE.  AN INITIAL VALUE MUST BE SUPPLIED LDA 1030
C                   BUT NEED NOT BE THE ONE WHICH WILL BE USED, SINCE THELDA 1040
C                   SUBROUTINE WILL CHOOSE A SMALLER ONE IF NECCESSARY TOLDA 1050
C                   KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED   LDA 1060
C                   VALUE.  IT IS BETTER TO UNDERESTIMATE THE INITIAL    LDA 1070
C                   STEPSIZE THAN TO OVERESTIMATE IT.  THE STEPSIZE IS   LDA 1080
C                   NORMALLY NOT CHANGED BY THE USER.                    LDA 1090
C        HMIN     - MINIMUM STEPSIZE ALLOWED                             LDA 1100
C        HMAX     - MAXIMUM STEPSIZE ALLOWED                             LDA 1110
C        RMSEPS   - THE ERROR TEST CONSTANT.  THE ROOT-MEAN-SQUARE OF    LDA 1120
C                   THE SINGLE STEP ERROR ESTIMATES,  ER(I), DIVIDED BY  LDA 1130
C                   YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I)) MUST BE  LDA 1140
C                   LESS THAN RMSEPS.  THE STEPSIZE AND/OR ORDER ARE     LDA 1150
C                   VARIED TO ACHIEVE THIS.                              LDA 1160
C        SAVE     - AN ARRAY OF LENGTH AT LEAST 7*NY                     LDA 1170
C        YLSV     - AN ARRAY OF LENGTH AT LEAST NL                       LDA 1180
C        YMAX     - A VECTOR OF LENGTH NY WHICH CONTAINS THE MAXIMUM     LDA 1190
C                   OF EACH Y SEEN SO FAR.  ON THE FIRST CALL, THESE WILLLDA 1200
C                   BE INITIALIZED AS YMAX(I) = MAX(1,|Y(1,I)|)          LDA 1210
C        ER       - A VECTOR OF LENGTH NY                                LDA 1220
C        ESV      - A VECTOR OF LENGTH NY                                LDA 1230
C        F1       - A VECTOR OF LENGTH N = NY + NL                       LDA 1240
C        DY       - A VECTOR OF LENGTH N = NY + NL                       LDA 1250
C        PW       - AN ARRAY IN WHICH THE  J  MATRIX COMPUTED            LDA 1260
C                   IN SUBROUTINE JACMAT WILL BE STORED.  SIZE WHICH     LDA 1270
C                   MUST BE ALLOWED IS DETERMINED BY THE STORAGE TECH-   LDA 1280
C                   NIQUE USED FOR IT, BUT NORMALLY WON'T BE MORE THAN   LDA 1290
C                   N**2 + 2*N LOCATIONS, THE LATTER 2*N BEING REQUIRED  LDA 1300
C                   BY THE LINEAR EQUATION SOLVER.                       LDA 1310
C                                                                        LDA 1320
C     -------------------------------------------------------------------LDA 1330
      DIMENSION Y(7,1), YL(1), SAVE(7,1), YMAX(1), ER(1), YLSV(1), F1(1)LDA 1340
     1, PERT(6,3), COF(21), ESV(1), DY(1), PW(1), SAV(1), A(29)          LDA 1350
      EQUIVALENCE (A(8),BND), (A(9),BR), (A(10),E), (A(11),EDWN),        LDA 1360
     1(A(12),ENQ1), (A(13),ENQ2), (A(14),ENQ3), (A(15),EPS), (A(16),EUP)LDA 1370
     2,(A(17),HNEW), (A(18),PEPSH), (A(19),IDOUB), (A(20),IWEVAL),       LDA 1380
     3(A(21),K), (A(22),LCOPYL), (A(23),LCOPYY), (A(24),MAXDER),         LDA 1390
     4(A(25),M1), (A(26),NL), (A(27),NQ), (A(28),NS), (A(29),NW)         LDA 1400
C                                                                        LDA 1410
C     -------------------------------------------------------------------LDA 1420
C                                                                        LDA 1430
C     THE COEFFICIENTS IN THE PERT ARRAY ARE USED FOR ERROR TESTING AND  LDA 1440
C     CHANGING STEPSIZE AND NEED TO BE ACCURATE TO ONLY A FEW DIGITS.    LDA 1450
```

```
C
C     -----------------------------------------------------------------
      DATA PERT/4.,9.,16.,25.,36.,49.,9.,16.,25.,36.,49.,64.,1.,1.,.25,
     12.7889E-2,1.70569E-3,6.83929E-5/
C     -----------------------------------------------------------------
C
C     THE ENTRIES IN THE COF ARRAY ARE THE COEFFICIENTS FOR THE STIFFLY
C     STABLE METHODS USED IN THIS PROGRAM AND ARE TO BE THE MACHINE
C     PRECISION EQUIVALENTS OF THE FOLLOWING CONSTANTS.
C
C     -1
C     -3/2 , -1/2
C     -11/6 , -1 , -1/6
C     -25/12 , -35/24 , -5/12 , -1/24
C     -137/60 , -15/8 , -17/24 , -1/8 , -1/120
C     -147/60, -203/90 , -49/48 , -35/144 , -7/240 , -1/720
C
C     -----------------------------------------------------------------
      DATA COF/-1.,-1.5,-.5,-1.833333,-1.,-.1666667,-2.083333,-1.458333,
     1-.4166667,-.04166667,-2.283333,-1.875,-.7083333,-.125,-.008333333,
     2-2.45,-2.255556,-1.020833,-.2430556,-.02916667,-.001388889/
      IF (JSTART) 100,110,150
C     -----------------------------------------------------------------
C     IF THIS IS A RESTART ENTRY, RESTORE Y AND YL FROM THE SAVE AND
C     YLSV ARRAYS, WHERE THEY WERE SAVED BY A PREVIOUS CALL TO LDASAV.
C     -----------------------------------------------------------------
  100 CALL COPYZ (Y,SAVE,LCOPYY)
      CALL COPYZ (YL,YLSV,LCOPYL)
      GO TO 150
C     -----------------------------------------------------------------
C     IF THIS IS THE FIRST CALL, INITIALIZE YMAX, SCALE DERIVATIVES, AND
C     INITIALIZE INDICATORS AND SET ORDER TO ONE.
C     FOR DOUBLE PRECISION, SET LCOPYL = 14*NY AND LCOPYL = 2*NL IF
C     SUBROUTINE COPYZ IS IN SINGLE PRECISION.
C     -----------------------------------------------------------------
  110 NL = N-NY
      LCOPYY = 7*NY
      LCOPYL = NL
      M1 = MIN0(M,NY)
      EPS = SQRT(FLOAT(M1))*RMSEPS
      MAXDER = MIN0(MAXOR,6)
      IF (IPRT.LE.0) GO TO 120
      PRINT 3, N,NL,RMSEPS,TEND,H
      PRINT 4
  120 NS = 0
      NW = 0
C
      DO 130 J=1,NY
```

```
      YMAX(J) = AMAX1(1.,ABS(Y(1,J)))
  130 Y(2,J) = Y(2,J)*H
C
      NQ = 1
      BR = 1.
      ASSIGN 190 TO IRET
C     -----------------------------------------------------------------------
C     SET COEFFICIENTS FOR THE ORDER CURRENTLY BEING USED.
C     E IS A TEST FOR ERRORS OF THE CURRENT ORDER NQ
C     EUP IS TO TEST FOR INCREASING THE ORDER, EDWN FOR DECREASING THE
C     ORDER.
C     -----------------------------------------------------------------------
  140 K = NQ*(NQ-1)/2
      CALL COPYZ (A(2),COF(K+1),NQ)
      K = NQ+1
      IDOUB = NQ
      ENQ1 = .5/NQ
      ENQ2 = .5/K
      ENQ3 = .5/(NQ+2)
      PEPSH = EPS**2
      E = PERT(NQ,1)*PEPSH
      EUP = PERT(NQ,2)*PEPSH
      EDWN = PERT(NQ,3)*PEPSH
      BND = (EPS*ENQ3)**2
      IWEVAL = 1
      GO TO IRET, (190,200,490,570)
  150 IF (H.EQ.HNEW) GO TO 190
C     -----------------------------------------------------------------------
C     IF CALLER HAS CHANGED H, RESCALE DERIVATIVES TO REFLECT THAT HNEW
C     WAS USED ON THE LAST CALL.
C     -----------------------------------------------------------------------
      R = H/HNEW
      ASSIGN 190 TO IRET
      GO TO 610
C     -----------------------------------------------------------------------
C     SET JSTART TO NQ, THE CURRENT ORDER OF THE METHOD, BEFORE EXIT,
C     AND SAVE THE CURRENT STEPSIZE IN HNEW.
C     -----------------------------------------------------------------------
  160 JSTART = NQ
      HNEW = H
      RETURN
  170 NS = NS+1
      IF (IPRT.LE.0) GO TO 180
C     -----------------------------------------------------------------------
C     PRINT DATA IF DESIRED BY USER
C     -----------------------------------------------------------------------
      PRINT 1, NS,NW,NQ,H,T,(Y(1,I),I=1,NY)
      IF (NL.GT.0) PRINT 2, (YL(I),I=1,NL)
```

```
  180 CONTINUE
      IF (KFLAG.LT.0) GO TO 160
      IF (T.GE.TEND) GO TO 160
C     -----------------------------------------------------------------
C     TAKE ANOTHER STEP IF T < TEND
C     -----------------------------------------------------------------
      JSTART = 1
C     -----------------------------------------------------------------
C     SAVE DATA FOR TRIAL WITH A SMALLER TIMESTEP IF THIS STEP FAILS
C     -----------------------------------------------------------------
  190 CALL COPYZ (SAVE,Y,LCOPYY)
      CALL COPYZ (YLSV,YL,LCOPYL)
      RACUM = 1.
      KFLAG = 1
      HOLD = H
      NQOLD = NQ
      TOLD = T
  200 T = T+H
      HINV = 1./H
C     -----------------------------------------------------------------
C     COMPUTE PREDICTED VALUES BY EFFECTIVELY MULTIPLYING DERIVATIVE
C     VECTOR BY PASCAL TRIANGLE MATRIX
C     -----------------------------------------------------------------
C
      DO 210 J=2,K
      J3 = K+J-1
C
      DO 210 J1=J,K
      J2 = J3-J1
C
      DO 210 I=1,NY
  210 Y(J2,I) = Y(J2,I)+Y(J2+1,I)
C
C
      DO 220 I=1,NY
  220 ER(I) = 0.
C
C     -----------------------------------------------------------------
C     DO UP TO THREE CORRECTOR ITERATIONS.  CONVERGENCE IS OBTAINED WHEN
C     CHANGES ARE LESS THAN BND WHICH IS DEPENDENT ON THE ERROR TEST
C     CONSTANT.  THE SUM OF CORRECTIONS IS ACCUMULATED IN ER(I).  IT IS
C     EQUAL TO THE K-TH DERIVATIVE OF Y TIMES H**K/(K-FACTORIAL*A(K)),
C     AND THUS IS PROPORTIONAL TO THE ACTUAL ERRORS TO THE LOWEST POWER
C     OF H PRESENT, WHICH IS H**K.
C     -----------------------------------------------------------------
C
      DO 270 L=1,3
      CALL DIFFUN (Y,YL,T,HINV,DY)
```

```
      IF (IWEVAL.LT.1) GO TO 230                                        LDA 2900
C     ------------------------------------------------------------------LDA 2910
C     IF THERE HAS BEEN A CHANGE OF ORDER OR THERE HAS BEEN TROUBLE     LDA 2920
C     WITH CONVERGENCE, PW IS RE-EVALUATED PRIOR TO STARTING THE        LDA 2930
C     CORRECTOR ITERATION.  IWEVAL IS THEN SET TO -1 AS AN INDICATOR    LDA 2940
C     THAT IT HAS BEEN DONE.  NEWPW IS SET NONZERO TO INDICATE TO       LDA 2950
C     SUBROUTINE NUITSL THAT A NEW PW HAS BEEN PROVIDED.                LDA 2960
C     ------------------------------------------------------------------LDA 2970
C     ------------------------------------------------------------------LDA 3250
      CALL JACMAT (Y,YL,T,HINV,A(2),N,NY,EPS,DY,F1,PW)                  LDA 2980
      KFLAG = 1                                                         LDA 2990
      IWEVAL = -1                                                       LDA 3000
      NW = NW+1                                                         LDA 3010
      NEWPW = 1                                                         LDA 3020
  230 CALL NUITSL (PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRRET)                  LDA 3030
      IF (KRRET.NE.0) GO TO 600                                         LDA 3040
      IF (NL.LE.0) GO TO 250                                            LDA 3050
C                                                                       LDA 3060
      DO 240 I=1,NL                                                     LDA 3070
  240 YL(I) = YL(I)-F1(I+NY)                                            LDA 3080
C                                                                       LDA 3090
  250 CONTINUE                                                          LDA 3100
      DEL = 0.                                                          LDA 3110
C                                                                       LDA 3120
      DO 260 I=1,NY                                                     LDA 3130
      Y(1,I) = Y(1,I)-F1(I)                                             LDA 3140
      Y(2,I) = Y(2,I)+A(2)*F1(I)                                        LDA 3150
      ER(I) = ER(I)+F1(I)                                               LDA 3160
      DEL = DEL+(F1(I)/AMAX1(YMAX(I),ABS(Y(1,I))))**2                   LDA 3170
  260 CONTINUE                                                          LDA 3180
C                                                                       LDA 3190
      IF (L.GE.2) BR = AMAX1(.9*BR,DEL/DEL1)                            LDA 3200
      DEL1 = DEL                                                        LDA 3210
      IF (AMIN1(DEL,BR*DEL*2.).LE.BND) GO TO 330                        LDA 3220
  270 CONTINUE                                                          LDA 3230
C                                                                       LDA 3240
C     THE CORRECTION ITERATION FAILED TO CONVERGE IN 3 TRIES.  VARIOUS  LDA 3260
C     POSSIBILITIES ARE CHECKED FOR.  IF H IS ALREADY HMIN AND PW HAS   LDA 3270
C     ALREADY BEEN RE-EVALUATED, A NO CONVERGENCE EXIT IS TAKEN.        LDA 3280
C     OTHERWISE THE MATRIX PW IS RE-EVALUATED AND/OR (IN THAT ORDER) THELDA 3290
C     STEP IS REDUCED TO TRY AND GET CONVERGENCE.                       LDA 3300
C     ------------------------------------------------------------------LDA 3310
      T = TOLD                                                          LDA 3320
      IF (IWEVAL) 280,300,290                                           LDA 3330
  280 IF (H.LE.HMIN*1.00001) GO TO 310                                  LDA 3340
  290 RACUM = RACUM*.25                                                 LDA 3350
  300 CONTINUE                                                          LDA 3360
      GO TO 560                                                         LDA 3370
```

```
  310 KFLAG = -3                                                    LDA 3380
C     ----------------------------------------------------------------LDA 3390
C     RESTORE Y AND YL AFTER CONVERGENCE FAILURE                    LDA 3400
C     ----------------------------------------------------------------LDA 3410
  320 CALL COPYZ (Y,SAVE,LCOPYY)                                    LDA 3420
      CALL COPYZ (YL,YLSV,LCOPYL)                                   LDA 3430
      H = HOLD                                                      LDA 3440
      NQ = NQOLD                                                    LDA 3450
      GO TO 170                                                     LDA 3460
C     ----------------------------------------------------------------LDA 3470
C     THE CORRECTOR CONVERGED, SO NOW THE ERROR TEST IS MADE.       LDA 3480
C     ----------------------------------------------------------------LDA 3490
  330 D = 0.                                                        LDA 3500
C                                                                   LDA 3510
      DO 340 I=1,M1                                                 LDA 3520
      YM = AMAX1(ABS(Y(1,I)),YMAX(I))                               LDA 3530
  340 D = D+(ER(I)/YM)**2                                           LDA 3540
C                                                                   LDA 3550
      IWEVAL = 0                                                    LDA 3560
      IF (D.GT.E) GO TO 380                                         LDA 3570
C     ----------------------------------------------------------------LDA 3580
C     THE ERROR TEST IS OKAY, SO THE STEP IS ACCEPTED.  IF IDOUB    LDA 3590
C     NOW BECOMES NEGATIVE, A TEST IS MADE TO SEE IF THE STEP SIZE  LDA 3600
C     CAN BE INCREASED AT THIS ORDER OR ONE HIGHER OR ONE LOWER.    LDA 3610
C     THE CHANGE IS MADE ONLY IF THE STEP CAN BE INCREASED BY AT    LDA 3620
C     LEAST 10%.  IDOUB IS SET TO NQ TO PREVENT FURTHER TESTING     LDA 3630
C     FOR A WHILE.  IF NO CHANGE IS MADE, IDOUB IS SET TO 9.        LDA 3640
C     ----------------------------------------------------------------LDA 3650
      IF (K.LT.3) GO TO 360                                         LDA 3660
C                                                                   LDA 3670
      DO 350 J=3,K                                                  LDA 3680
C                                                                   LDA 3690
      DO 350 I=1,NY                                                 LDA 3700
  350 Y(J,I) = Y(J,I)+A(J)*ER(I)                                    LDA 3710
C                                                                   LDA 3720
  360 KFLAG = 1                                                     LDA 3730
      IDOUB = IDOUB-1                                               LDA 3740
      IF (IDOUB) 410,370,510                                        LDA 3750
  370 CALL COPYZ (ESV,ER,M1)                                        LDA 3760
      GO TO 510                                                     LDA 3770
C     ----------------------------------------------------------------LDA 3780
C     THE ERROR TEST FAILED.  IF JSTART = 0, THE DERIVATIVES IN THE LDA 3790
C     SAVE ARRAY ARE UPDATED.  TESTS ARE THEN MADE TO FIX THE STEPSIZE LDA 3800
C     AND PERHAPS REDUCE THE ORDER.  AFTER RESTORING AND SCALING THE LDA 3810
C     Y VARIABLES, THE STEP IS RETRIED.                             LDA 3820
C     ----------------------------------------------------------------LDA 3830
C     ----------------------------------------------------------------LDA 4300
  380 IF (JSTART.GT.0) GO TO 400                                    LDA 3840
```

```
C
      DO 390 I=1,NY
  390 SAVE(2,I) = Y(2,I)
C
  400 KFLAG = KFLAG-2
      IF (H.LE.HMIN) GO TO 550
      T = TOLD
      IF (KFLAG.LE.-5) GO TO 530
  410 PR2 = (D/E)**ENQ2*1.2
      L = 0
      IF (NQ.LE.1) GO TO 430
      D = 0.
C
      DO 420 J=1,M1
      YM = AMAX1(ABS(Y(1,J)),YMAX(J))
  420 D = D+(Y(K,J)/YM)**2
C
      PR1 = (D/EDWN)**ENQ1*1.3
      IF (PR1.GE.PR2) GO TO 430
      PR2 = PR1
      L = -1
  430 IF (KFLAG.LT.0.OR.NQ.GE.MAXDER) GO TO 450
      D = 0
C
      DO 440 J=1,M1
      YM = AMAX1(ABS(Y(1,J)),YMAX(J))
  440 D = D+((ER(J)-ESV(J))/YM)**2
C
      PR1 = (D/EUP)**ENQ3*1.4
      IF (PR1.GE.PR2) GO TO 450
      PR2 = PR1
      L = 1
  450 R = 1./AMAX1(PR2,1.E-5)
      IF (KFLAG.LT.0.OR.R.GE.1.1) GO TO 460
      IDOUB = 9
      GO TO 510
  460 NEWQ = NQ+L
      K = NEWQ+1
      IF (NEWQ.LE.NQ) GO TO 480
      R1 = A(NEWQ)/FLOAT(NEWQ)
C
      DO 470 J=1,NY
  470 Y(K,J) = ER(J)*R1
C
  480 CONTINUE
C     IF THE STEP WAS OKAY, SCALE THE Y VARIABLES IN ACCORDANCE
C     WITH THE NEW VALUE OF H.  IF KFLAG < 0, HOWEVER, USE THE
C     SAVED VALUES (IN SAVE AND YLSV).  IN EITHER CASE, IF THE ORDER
```

```
C       HAS CHANGED IT IS NECESSARY TO FIX CERTAIN PARAMETERS BY CALLING  LDA 4340
C       THE PROGRAM SEGMENT AT STATEMENT NUMBER 140.                      LDA 4350
C       -----------------------------------------------------------------LDA 4360
        IDOUB = NQ                                                        LDA 4370
        IF (NEWQ.EQ.NQ) GO TO 490                                         LDA 4380
        NQ = NEWQ                                                         LDA 4390
        ASSIGN 490 TO IRET                                                LDA 4400
        GO TO 140                                                         LDA 4410
  490   IF (KFLAG.GT.0) GO TO 500                                         LDA 4420
        RACUM = RACUM*R                                                   LDA 4430
        GO TO 560                                                         LDA 4440
  500   R = AMAX1(AMIN1(HMAX/H,R),HMIN/H)                                 LDA 4450
        H = H*R                                                           LDA 4460
        IWEVAL = 1                                                        LDA 4470
        ASSIGN 510 TO IRET                                                LDA 4480
        GO TO 610                                                         LDA 4490
C                                                                         LDA 4500
  510   DO 520 I=1,M1                                                     LDA 4510
  520   YMAX(I) = AMAX1(ABS(Y(1,I)),YMAX(I))                              LDA 4520
C                                                                         LDA 4530
        GO TO 170                                                         LDA 4540
C       -----------------------------------------------------------------LDA 4550
C       THE ERROR TEST HAS NOW FAILED THREE TIMES, SO THE DERIVATIVES ARE LDA 4560
C       IN BAD SHAPE.  RETURN TO FIRST ORDER METHOD AND TRY AGAIN.  OF    LDA 4570
C       COURSE, IF NQ = 1 ALREADY, THEN THERE IS NO HOPE AND WE EXIT WITH LDA 4580
C       KFLAG = -4.                                                       LDA 4590
C       -----------------------------------------------------------------LDA 4600
  530   IF (NQ.EQ.1) GO TO 540                                            LDA 4610
        NQ = 1                                                            LDA 4620
        IDOUB = 1                                                         LDA 4630
        ASSIGN 570 TO IRET                                                LDA 4640
        GO TO 140                                                         LDA 4650
  540   NQOLD = 1                                                         LDA 4660
        KFLAG = -4                                                        LDA 4670
        GO TO 320                                                         LDA 4680
  550   KFLAG = -1                                                        LDA 4690
        GO TO 170                                                         LDA 4700
C       -----------------------------------------------------------------LDA 4710
C       THIS SECTION RESTORES THE SAVED VALUES OF Y AND YL, SCALING THE   LDA 4720
C       Y DERIVATIVES AS NECESSARY, AND THEN RETURNS TO THE PREDICTOR LOOPLDA 4730
C       -----------------------------------------------------------------LDA 4740
  560   H = HOLD*RACUM                                                    LDA 4750
        H = AMAX1(HMIN,AMIN1(H,HMAX))                                     LDA 4760
  570   RACUM = H/HOLD                                                    LDA 4770
        R1 = 1.                                                           LDA 4780
C                                                                         LDA 4790
        DO 580 J=2,K                                                      LDA 4800
        R1 = R1*RACUM                                                     LDA 4810
```

```
C
      DO 580 I=1,NY
  580 Y(J,I) = SAVE(J,I)*R1
C
C
      DO 590 I=1,NY
  590 Y(1,I) = SAVE(1,I)
C
      CALL COPYZ (YL,YLSV,LCOPYL)
      IWEVAL = 1
      GO TO 200
  600 KFLAG = -5
      GO TO 160
C     -----------------------------------------------------------------
C     THIS SECTION SCALES THE Y DERIVATIVES BY R**J
C     -----------------------------------------------------------------
  610 R1 = 1.
C
      DO 620 J=2,K
      R1 = R1*R
C
      DO 620 I=1,NY
  620 Y(J,I) = Y(J,I)*R1
C
      GO TO IRET, (190,510)
C     -----------------------------------------------------------------
C     THIS SECTION ALLOWS FOR RESTARTS AFTER SOLVING ANOTHER PROBLEM, OR
C     HAVING TERMINATED THE CURRENT COMPUTER RUN.  SUBROUTINE LDASAV
C     SAVES THE NECESSARY VALUES WHICH ARE INTERNAL TO LDASUB.  FOR
C     DOUBLE PRECISION, WITH COPYZ IN SINGLE PRECISION, THE NUMBER OF
C     LOCATIONS TO BE SAVED AND RESTORED, LCOPYS AND LCOPYR, MUST BE
C     SET TO 58.
C     IT IS ASSUMED THAT IN ADDITION TO THE VARIABLES IN THE ARRAY A
C     SAVED BY CALLING LDASAV, THE USER ALSO SAVES THE ARRAYS SAVE,
C     YLSV, YMAX, ESV, AND PW.
C
C     TO RESTART THE USER FIRST CALLS LDARST TO RESTORE THE VALUES SAVED
C     BY LDASAV, THEN RE-ENTERS LDASUB WITH JSTART < 0, AND WITH THE
C     OTHER PARAMETERS THE SAME AS RETURNED FROM THE LAST ENTRY TO
C     LDASUB, PARTICULARLY THOSE ARRAYS MENTIONED ABOVE.
C     -----------------------------------------------------------------
      ENTRY LDASAV(SAV)
      LCOPYS = 29
      CALL COPYZ (SAV,A,LCOPYS)
      CALL COPYZ (SAVE,Y,LCOPYY)
      CALL COPYZ (YLSV,YL,LCOPYL)
      RETURN
C
```

```
      ENTRY LDARST(SAV)
      LCOPYR = 29
      CALL COPYZ (A,SAV,LCOPYR)
      RETURN
C
C     ------------------------------------------------------------------
C
C
    1 FORMAT (2I5,I2,1P2E10.2,7E14.6/(32X,7E14.6))
    2 FORMAT (32X,1P7E14.6)
    3 FORMAT ('1  N =',I3,'   NL =',I3,'   RMSEPS =',1PE9.2,'   TEND ='
     1  ,E9.2,'   H =',E9.2//)
    4 FORMAT ('   NS   NW Q    H',8X,'T    ',8X,'Y(1,*) AND YL(*)'//)
      END
C
      SUBROUTINE COPYZ(S,Y,L)
      DIMENSION S(1),Y(1)
C     ------------------------------------------------------------------
C
C     THIS SUBROUTINE COPIES THE ARRAY Y, OF LENGTH L, INTO THE ARRAY S
C
C     ------------------------------------------------------------------
      IF(L.LE.0)RETURN
      DO 100 J=1,L
  100 S(J) = Y(J)
      RETURN
      END
C
      SUBROUTINE DERVAL (Y,YL,T,N,NY,W,KERET)
C
C     THIS SUBROUTINE CALCULATES THE INTIAL VALUES OF THE DERIVATIVES
C     IN THE GENERAL CASE.  IT IS WRITTEN SO THAT IT SHOULD WORK IF THE
C     FIRST NY EQUATIONS ALL INVOLVE DERIVATIVES.  IT ATTEMPTS TO SOLVE
C     THE FIRST NY EQUATIONS USING NEWTON'S METHOD, BUT SINCE IT TRIES
C     TO EVALUATE DF/DY' BY CALLING JACMAT IN SUCH A WAY AS TO MAKE THE
C     DF/DY TERM INSIGNIFICANT, IT IS POSSIBLE THAT IT MAY FAIL FOR THAT
C     REASON.  IT MAY FAIL FOR OTHER REASONS, AS WELL.  IF IT DOES FAIL
C     THE USER CAN SUPPLY HIS OWN VERSION OF DERVAL, OR MODIFY THIS
C     ROUTINE IN SUITABLE FASHION.  THIS ROUTINE ASSUMES THAT VALUES OF
C     THE LINEAR VARIABLES HAVE BEEN SUPPLIED PREVIOUSLY.  IF THOSE
C     MUST BE SOLVED FOR SIMULTANEOUSLY WITH THE DERIVATIVES, THE USER
C     MUST SUPPLY HIS OWN VERSION OF DERVAL.
C
C     THE CALLING SEQUENCE FOR THIS SUBROUTINE IS
C
C     CALL DERVAL(Y,YL,T,N,NY,W,KERET)
C
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS
```

```
C
C        Y        - SAME AS IN LDASUB AND SDESOL.  Y(1,I) CONTAINS THE
C                    INITIAL VALUES OF THE DEPENDENT VARIABLES.    THE
C                    VALUES OF THE DERIVATIVES ARE RETURNED IN Y(2,I).
C        YL       - SAME AS IN LDASUB AND SDESOL.  THE INITIAL VALUES OF
C                    THE LINEAR VARIABLES MUST BE SUPPLIED TO THIS VERSION
C        T        - INITIAL TIME
C        N        - SAME AS IN LDASUB, TOTAL NUMBER OF VARIABLES
C        NY       - SAME AS IN LDASUB, NUMBER OF DIFFERENTIAL EQUATIONS
C                    AND NONLINEAR VARIABLES
C        W        - SCRATCH ARRAY W FROM THE CALLING SEQUENCE OF SDESOL.
C                    THIS CAN BE USED AS NEEDED IN THIS SUBROUTINE.
C        KERET    - RETURN INDCATOR
C                         =0   NORMAL RETURN
C                         =1   ERROR RETURN
C
C     -----------------------------------------------------------------
      DIMENSION Y(7,1), YL(1), W(1)
C
      DO 100 I=1,NY
      W(2*N+I) = AMAX1(ABS(Y(1,I)),1.)
  100 Y(3,I) = 0.
C
      HINV = 16.**20
      KERET = 0
      EPS2 = NY/1.E8
      EPS = SQRT(EPS2)
C
C
      DO 140 IT=1,10
      DO 110 I=1,NY
  110 Y(2,I) = Y(3,I)/HINV
C
      CALL DIFFUN (Y,YL,T,HINV,W)
      CALL JACMAT (Y,YL,T,HINV,-1.,NY,NY,EPS,W,W(N+1),W(3*N+1))
      NEWPW = 1
C
      DO 120 I=1,NY
  120 W(I) = W(I)*HINV
C
      CALL NUITSL (W(3*N+1),W,W(N+1),NY,NY,EPS,W(2*N+1),NEWPW,KRET)
      IF (KRET.NE.0) GO TO 170
      ER = 0.
C
      DO 130 I=1,NY
      Y(3,I) = Y(3,I)-W(N+I)
  130 ER = ER+(W(N+I)/AMAX1(ABS(Y(3,I)),1.))**2
C
```

```
      IF (ER.LT.EPS2) GO TO 150                                         DER  690
  140 CONTINUE                                                          DER  700
C                                                                       DER  710
      GO TO 170                                                         DER  720
C                                                                       DER  730
  150 DO 160 I=1,NY                                                     DER  740
  160 Y(2,I) = Y(3,I)                                                   DER  750
C                                                                       DER  760
      RETURN                                                            DER  770
  170 KERET = 1                                                         DER  780
      RETURN                                                            DER  790
      END                                                               DER  800
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
C     MUST BE SUPPLIED BY THE JSER. ITS PURPJSE IS TO EVALUATE THE
C     FUNCTION AT CURRENT VALUES OF THE VARIABLES.
C-----------------------------------------------------------------------
C
C
C
      SUBROUTINE DIFFUN(Y,YL,T,HINV,DY)                                  D-005
      REAL*8 C1,C2,C4,C5
      REAL*8 BIGG,BIGGG,BIGH                                             D-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN                D-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF, D-020
     1CONN(128,15)                                                       D-025
      DIMENSION Y(7,1),YL(1),DY(1)                                       D-035
      COMMON/DELAY/SUM(183),PSI(93)
      COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
      COMMON/SYSMT1/BIGG(4615)                                           D-050
      COMMON/SYSMT2/BIGGG(4615)                                          D-055
      COMMON/SYSMT3/BIGH(4615)
      DATA TOLD/-1.0E-30/                                                D-065
      IF(T.EQ.TOLD) GO TO 97                                             D-075
      CALL SUMT(Y,TOLD,T-TOLD)                                           D-080
   96 TOLD=T                                                             D-085
   97 CONTINUE                                                           D-090
      CALL FEEDBK(Y,TOLD)                                                D-100
      DO 110 I=1,NNZ                                                     D-110
      IF(I.GT.93) GO TO 401
      PSI(I)=Y(1,I)                                                      D-115
  401 CONTINUE
      DY(I)=0.0                                                          D-120
      JBB=JB(I)                                                          D-125
      JE=JB(I)+JA(I)-1                                                   D-130
      DO 120 J=JBB,JE                                                    D-135
      LL=NAME(J)                                                         D-140
```

```
      IF(LL.GT.NNZ) GO TO 120                                           D-145
      DY(I)=DY(I)+BIGG(J)*(HINV*Y(2,LL)+C2(LL)*Y(1,LL)+C5(LL)*SUM(LL))+C
     11(LL)*BIGGG(J)*Y(1,LL)+C4(LL)*BIGH(J)*Y(1,LL)
  120 CONTINUE                                                          D-180
  110 CONTINUE                                                          D-185
      RETURN                                                            D-195
      END                                                               D-200
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
C     MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO EVALUATE THE J
C     MATRIX NEEDED WHEN THE CORRECTOR EQUATION IS BEING SOLVED.
C-----------------------------------------------------------------------
C
C
      SUBROUTINE JACMAT(Y,YL,T,HINV,A2,N,NY,EPS,DY,F1,PW)               J-005
      REAL*8 C1,C2,C4,C5
      REAL*8 BIGG,BIGGG,BIGH                                            J-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN               J-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF, J-020
     1CONN(128,15)                                                      J-025
      DIMENSION Y(7,1),YL(1),F1(1),DY(1),PW(1)                          J-035
      COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
      COMMON/SYSMT1/BIGG(4615)                                          J-045
      COMMON/SYSMT2/BIGGG(4615)                                         J-050
      COMMON/SYSMT3/BIGH(4615)
      AH =-A2*HINV                                                      J-065
      DO 300 I=1,NNZ                                                    J-075
      JBB=JB(I)                                                         J-080
      JE=JB(I)+JA(I)-1                                                  J-085
      DO 310 J=JBB,JE                                                   J-090
      LL=NAME(J)
      PW(J)=BIGG(J)*(AH+C2(LL))+C1(LL)*BIGGG(J)+C4(LL)*BIGH(J)
  310 CONTINUE                                                          J-115
  300 CONTINUE                                                          J-120
      RETURN                                                            J-130
      END                                                               J-135
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
C     MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO SOLVE A LINEAR
C     SYSTEM OF EQUATIONS FOR THE NEWTON ITERATES WHEN THE CORRECTOR
C     EQUATION IS BEING SOLVED.
C-----------------------------------------------------------------------
C
C
      SUBROUTINE NUITSL(PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRET)              N-005
      INTEGER*2 K,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN                  N-010
```

```
      COMMON/GTRY2/K(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,CON
     1N(128,15)
      DIMENSION PW(1),DY(1),F1(1),YMAX(1)
      DATA  SPD,SPDM1/1.05,.05/
      KRET = 0
      EPSS = EPS**2
      EPSA2 = EPSS*.0001
      NOIT = N
  280 DO 281 I=1,NY
      LL=JB(I)
  281 F1(I)=DY(I)/PW(LL)
      DO 287 IT=1,NOIT
      RCH = 0.
      CH = 0.
      DO 285 I=1,NY
      LL=JB(I)
      JBB=JB(I)+1
      JE=JB(I)+JA(I)-1
      FN = DY(I)
      DO 284 J=JBB,JE
      IF(K(J).LE.0.OR.K(J).GT.NY) GO TO 284
      FN=FN-PW(J)*F1(K(J))
  284 CONTINUE
      FN=FN/PW(LL)
      FN = FN*SPD - SPDM1*F1(I)
      ACH = F1(I) - FN
      CH = CH + (ACH/YMAX(I))**2
  282 RCH = RCH + (ACH/AMAX1(ABS(FN),EPS))**2
  285 F1(I) = FN
      IF(RCH.LT.EPSS)GO TO 288
      IF(CH.LE.EPSA2)GO TO 288
  287 CONTINUE
      KRET = 3
  288 CONTINUE
      RETURN
      END
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE CALCULATES THE GGG ELEMENT MATRIX USING 21 POINTS
C     OF INTEGRATION. IT THEN PUTS THE GGG ELEMENT MATRIX INTO THE BIGH
C     SYSTEM MATRIX.
C-----------------------------------------------------------------------
C
C
      SUBROUTINE FEEDBK(Y,TOLD)
      REAL*8 GGG,BIGH
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN
```

```
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,   T-025
     1CONN(128,15)                                                         T-030
      COMMON/GTRY1/X(505),P(505),Z(505)                                    T-035
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)                           T-045
      DIMENSION Y(7,1)                                                     T-040
      COMMON/TEMP/C3(183),C6(15)
      COMMOV/GGGMAT/GGG(15,15)                                             T-055
      COMMON/SYSMT3/BIGH(4615)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1   T-065
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)                       T-070
      DO 300 I=1,JC
      BIGH(I)=0.0                                                          T-085
  300 CONTINUE                                                             T-090
      NNAAA=7                                                              T-095
      NNSSS=3                                                              T-100
      DO 800 L=1,NUMEL
      DO 600 I=1,15                                                        T-120
      DO 610 J=1,15                                                        T-125
      GGG(I,J)=0.0                                                         T-130
  610 CONTINUE                                                             T-135
  600 CONTINUE                                                             T-140
      DO 805 JJ=1,15                                                       T-145
      NN=CONN(L,JJ)                                                        T-155
      XX(JJ)=X(NN)                                                         T-160
      YY(JJ)=P(NN)                                                         T-165
      ZZ(JJ)=Z(NN)                                                         T-170
      IF(NN.GT.NNZ) GO TO 803                                              T-175
      C6(JJ)=C3(NN)
      PPSI(JJ)=Y(1,NN)                                                     T-180
      GO TO 805                                                            T-185
  803 PPSI(JJ)=0.0                                                         T-190
      C6(JJ)=0.0
  805 CONTINUE                                                             T-195
      A5=2.0/(ZZ(1)-ZZ(15))                                                T-200
      DETJJ=0.0                                                            T-205
      DO 802 K=1,NNSSS                                                     T-210
      DETJ1=0.0                                                            T-215
      DO 798 M=1,NNAAA                                                     T-220
      IF(K.EQ.1) GO TO 111                                                 T-225
      IF(K.EQ.2) GO TO 222                                                 T-230
      IF(K.EQ.3) GO TO 333                                                 T-235
      GO TO 788                                                            T-240
  111 N=M                                                                  T-245
      GO TO 788                                                            T-250
  222 N=M+7                                                                T-255
      GO TO 788                                                            T-260
  333 N=M+14                                                               T-265
  788 CONTINUE                                                             T-270
```

```
      T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)       T-275
      T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)   T-280
      T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)       T-285
      T4=(DL1(15,N)-DL3(15,N))*XX(15)                                       T-290
      DJ11=T1+T2+T3+T4                                                      T-295
      D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)       T-300
      D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)   T-305
      D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)       T-310
      D4=(DL1(15,N)-DL3(15,N))*YY(15)                                       T-315
      DJ12=D1+D2+D3+D4                                                      T-320
      T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)       T-325
      T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)           T-330
      T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)    T-335
      T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)                                 T-340
      DJ21=T5+T6+T7+T8                                                      T-345
      D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)       T-350
      D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)           T-355
      D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)    T-360
      D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)                                 T-365
      DJ22=D5+D6+D7+D8                                                      T-370
      DETJ(N)=DJ11*DJ22-DJ12*DJ21                                           T-375
      DETJ1=DETJ(N)*WL(M)+DETJ1                                             T-380
  798 CONTINUE                                                              T-385
      DETJJ=DETJJ+WS(K)*DETJ1                                               T-390
  802 CONTINUE                                                              T-395
      DO 700 J=1,15                                                         T-400
      DO 710 I=1,15                                                         T-405
      FHH=0.0                                                               T-410
      DO 806 K=1,NNSSS                                                      T-415
      FH=0.0                                                                T-420
      DO 799 M=1,NNAAA                                                      T-425
      TT1=0.0                                                               T-430
      TT2=0.0                                                               T-435
      TT3=0.0                                                               T-440
      IF(K.EQ.1) GO TO 1111                                                 T-445
      IF(K.EQ.2) GO TO 2222                                                 T-450
      IF(K.EQ.3) GO TO 3333                                                 T-455
      GO TO 7888                                                            T-480
 1111 N=M                                                                   T-485
      GO TO 7888                                                            T-490
 2222 N=M+7                                                                 T-495
      GO TO 7888                                                            T-500
 3333 N=M+14                                                                T-505
 7888 CONTINUE                                                              T-550
  770 TT1=FN(1,N)*PPSI(1)*C6(1)+FN(2,N)*PPSI(2)*C6(2)+FN(3,N)*PPSI(3)*C6
     1(3)+FN(4,N)*PPSI(4)*C6(4)+FN(5,N)*PPSI(5)*C6(5)+FN(6,N)*PPSI(6)*C6
     1(6)+FN(7,N)*PPSI(7)*C6(7)+FN(8,N)*PPSI(8)*C6(8)
      TT2=FN(9,N)*PPSI(9)*C6(9)+FN(10,N)*PPSI(10)*C6(10)+FN(11,N)*PPSI(1
```

```
     11)*C6(11)+FN(12,N)*PPSI(12)*C6(12)+FN(13,N)*PPSI(13)*C6(13)+FN(14,
     1N)*PPSI(14)*C6(14)+FN(15,N)*PPSI(15)*C6(15)
      TT3=TT1+TT2+1.0
      FH=FH+WL(M)*FN(J,N)*FN(I,N)*ALOG(TT3)*DETJ(N)
  799 CONTINUE
      FHH=WS(K)*FH+FHH
  806 CONTINUE
      GGG(J,I)=FHH/45
  710 CONTINUE
  700 CONTINUE
C     INSERT INTO SYSTEM MATRIX
      DO 20 K=1,15
      KK=CONN(L,K)
      IF(KK.GT.NNZ) GO TO 20
      KKK=JA(KK)
      LLL=JB(KK)-1
      DO 10 I=1,15
      II=CONN(L,I)
      DO 91 M=1,KKK
      MM=LLL+M
      KKM=NAME(MM)
      IF(II.EQ.KKM)GO TO 92
   91 CONTINUE
      GO TO 10
   92 CONTINUE
      BIGH(MM)=BIGH(MM)+GGG(K,I)
   10 CONTINUE
   20 CONTINUE
  800 CONTINUE
      RETURN
      END
C
C---------------------------------------------------------------------
C     THE CUMULATIVE CONTRIBUTION OF THE DELAYED NEUTRON FLUX IS
C     CALCULATED BY THIS SUBROUTINE.
C---------------------------------------------------------------------
C
      SUBROUTINE SUMT(Y,T,H)
      DIMENSION Y(7,1)
      COMMON/DELAY/SUM(183),PSI(93)
      Q=0.4349710
      TC=-Q*H
      DO 100 I=1,93
      SUM(I)=SUM(I)*EXP(-Q*H)+0.50*H*(PSI(I)*EXP(TC)+Y(1,I))
  100 CONTINUE
      RETURN
      END
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LIST OF REFERENCES

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